A Study of Some Problems Relating to Blood Flow in Human Circulatory System

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Certificate

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Rajan Singh

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P R E F A C E

The present thesis entitled "A study of some problems relating to blood flow in human circulatiory system", is a record of genuine research work carried out by me under the supervision and guidance of Dr. J.S. Parihar (Supervisor), Reader in the Deptt. of Maths, Atarra P.G. College Atarra (Banda)

The purpose of the thesis is to study some problems on blood flow in human circulatory system in those vessels whose diameters are very small less than 500 μm, $1 \mu m = 10^{-6}$m.

The effects of peripheral layer, yield stress and other rheological Parameters on blood flow have been analysed. We have studied the problems on steady flow

of blood, effects of mild stenosis on blood flow through narrow vessels, Blood flow in narrrow tapered tube. Flow of Bingham fluid in capillaries, Pulsatile blood flow through narrow vessels.

This thesis consist of seven chapters. Chapter I contains the general intrduction of the subject (viz. circulatory system, rheology of blood geometrical aspect of vessels etc.) and a brief account of the relevant literature.

In chapter II, rheological behaviour of steady blood flow in narrow vessels have been studied. Two layers flow model, the central core layer and peripheral layer, satisfing herschel Bukley constitutive equation and casson equation have been considered for the analysis. Velocity profile and flow rate have been discussed with respect to the peripheral layer thickness, yield stress and parameter n.

From the graph we find that as $\bar{\tau}_0$ increases (μ remaining) the same) the flux rate decreases rapidly till Cp = .6 and till it has fallen to about 5 percent of Q_0 and then it rises again. Small changes in $\bar{\tau}_0$ can make significant changes in Q.

In Chapter III, a study of casson fluid model of blood flowing through a narrow tapered tube has been

done. Two layers flow model, the central core layer and peripheral layer satisfying casson constitutive equation have been studied for tapered narrow tube. Velocity profite and flow rate have been discussed with the help of tables and graphs.

The variations of pressure gradient and shear stress at the wall have been computed for different tapered angles anddifferent axial distances. it is observed that the pressure gradient increases as the tapered angle and axial distanee increases. The variationof shear stressat the wall with flow rate for different tapered angles have been studied. We find that τ_w increases with the increase in suspension concentration and tapered angle.

In chapter IV effect of mild stenosis on blood flow throw narrow vessels has been studied. In this chaper a non Newtonian Heschel - Bulkley fluid model has been studied in the presence of mild stenosis. Analytical expressions for volume flow rate, apparent fluidity and wall shear stress at the maxumum height of stenosis have been obtained.

In this chapter we study variation of volume flow rate (Q) with stenosis height $\left(\frac{\sigma}{R_0}\right)$ and yield stress

(β) for different values of n (=0.5,1.0,1.5)

Graphs show that value of Q decreases as $\frac{\sigma}{R_0}$ and β increases for fixed value of n. From results it is observed that the flow rate is decreased from Newtonian case by 25% for n=.5, 13% for n=1; 7% for n=1.5 for fixed $\frac{\sigma}{R_0}$ and β. For n= .10, the value of $\bar{Q}$ are decreased by 30% from those for no stenosis case for fixed n and β.

In chapter V, a study of Bingham fluid model of blood flow in capillaries has been performed. In this chapter a two layer fluid model of blood with no slip velocity at the wall both satisfying Bingham constitutive equation is considered. In this chapter velocity field, volume flow rate and apparent fluidity have been found. Flow parameters have been explained. Results show that apparent fluidity fa, increases with d (plasma layer thickness) and decreases with α_c (yield stress). It is also observed that velocity field decreases fastly with r in the peripheral layer whereas in the core region velocity decreases slowly.

In chapter VI a theoretical study of pulsatile blood flow through narrow vessels is performed. In this chapter we have analysed the pulsatile blood flows throw straight and long narrow rigid circular tube

whose radius is constant. It is assumed that flow of blood is axially symmetric and in the z derection. In this chapter, the characteristics discussed are the pressure, shear stress at the wall, flow rate and viscosity. It is seen that the effect of frequency parameter α on velocity, shear stress, plug radius and flow radius is very small in the range $0 \le \alpha \le 1$.

It is also found that values of R_p, $\bar{\tau}_w$ and Q (t) vary greatly with respect to time specially for large values of reduced amplitude A.

In Chapter VII, a theoretical model of steady blood flow in narrow vessel has been performed. Inthis chapter it has been assumed that fluid in core region satisfy casson equation and in mari\ginal layer region satisfy Newtonian equation. Apparent viscosity of blood has been determined as a function of yield stress, vessel diameter and peripheral layer (PPL) thickness.

Here we have taken the data of Charme and Kurland and shown the variation of μ_{app} with constant in a table, assuming that the casson vicosity is related by general equation -

$$\mu_c = \mu_p (1 - KH)^{-1}$$

Where H is function of hematicrit and K is an expirimental constant, We can obtain μ_{app} atdifferent concertration of Hematocrit.

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Where H is function of hematocrit and K is an expirimertal constant, We can obtain μ_{app} at different concertration of Hematocrit.

CONTENTS

CHAPTER I
General Introduction

1.1 Introduction

Biomechanics is the field which has been developed recently by cross fertilization of the mechanics and biological sciences, so that both are being profited and are utilized for the benefit of mankind.

Biomechanics is the study where in the principles of fluid mechanics are used to understand the problems in biology.

In the present study we concentrate to the study of blood rheology and blood flow in human circulatory system.

1.2 Navier - Stokes Equations for the Flow of Viscous Incompressible Fluid

Let u (x, y, z, t), v (x, y, z, t), w (x, y, z, t) and p (x, y, z, t) denote respectively the three velocity components and pressure at the point (x, y, z) at time t in a fluid with constant density ρ and viscosity coefficient μ. Then the equation of continuity, which expresses the fact that the amount of fluid entering a unit volume per unit time is the same as the amount of the fluid leaving it per unit time, is given by -

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

The equation of motion are obtained from Newtons second law of motion which states that the product of mass and acceleration of any fluid is equal to the resultant of all the external body forces acting on the element and to the surface forces acting on the fluid volume due to the action of the remaining fluid on the same element. The equations of motion, known as Navier stokes equations for the flow of a Newtonian viscous incompressible fluid are -

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \qquad (2)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \qquad (3)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \qquad (4)$$

If the external body forces x, y, z form a conservative system, there exists a potential function Ω such that

$$x=-\frac{\partial\Omega}{\partial x},\quad y=-\frac{\partial\Omega}{\partial y},\quad z=-\frac{\partial\Omega}{\partial z},$$

$$x-\frac{\partial p}{\partial x}=-\frac{\partial}{\partial x}(\Omega+p),\quad y-\frac{\partial p}{\partial y}=\frac{-\partial}{\partial y}(\Omega+p),$$

$$z-\frac{\partial p}{\partial z}=\frac{-\partial}{\partial z}(\Omega+p),$$

So that P is effectively replaced by P + Ω

1.3 Reynold Number of Flows -

In equations (2) to (4) the terms on the left hand sides represent the inertial forces (mass x acceleration) while the three terms on the right hand side of each equation represent respectively

the body forces, pressure forces and viscous forces. If U is a typical velocity and L is a typical length the inertial forces are of the order $\rho U^2 / L$ and the viscous forces are of the order $\mu U / L^2$. The radio of these forces is of the order

$$Re = \frac{\rho U^2 / L}{\mu U / L^2} = \frac{\rho U^2 L^2}{L \mu U} = \frac{\rho UL}{\mu} = \frac{UL}{v}$$

Where $v = \frac{\mu}{\rho}$ is called the kinematic viscocity of the fluid. Now the diamensions of μ and ρUL are given by-

$$\mu = \frac{Stress}{Strainrate} = \frac{Force.\ per.\ unit.\ area}{Velocity / lenght}$$

$$= \frac{MLT^{-2} / L^2}{T^{-1}} = ML^{-1}T^{-1} \quad (5)$$

$$\rho UL = ML^{-3}LT^{-1}L = ML^{-1}T^{-1} \quad (6)$$

Thus Re is a dimensionless number. It is called Reynold's number, after asborn Reynold who in 1890 showed that the fully developed Poiseuille flow in a circular tube changes from stream line or laminar flow to turbulent flow when this number, based on the diameter of the tube, exceeds a critical value of about 2000.

In straight pipes, turbulence occurs when Re exceeds 2000; in curved pipes it may not occur even for Re = 6000.

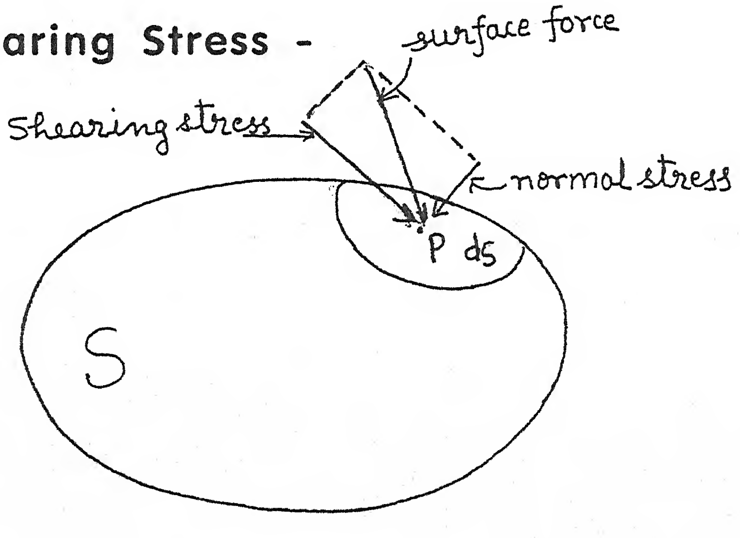
When Reynold number is small, viscous forces dominate over inertial forces.

1.4 Basic Concepts about Blood, Cardiovascular system and Blood Flows -

Constitution of Blood -

Blood consists of a suspension of cells in an aqueous solution called plasma which is composed of about 90 percent water and 7% protein. There are about 5×10^9 cells in a millilitre (1cc) of healthy human blood, of which about 95 percent are red cells or erythrocytes whose main function is to transport oxygen from the lungs to all the cells of the body and the removal of carbondioxide formed by metabolic processes in the body to the lungs. About 45 percent of the blood volume in an average man is occupied by red cells. This fraction is known as the hematocrit, Of the remaining, white cells or leucocytes constitute about one sixth or 1 percent of the total, and these play a role in the resistance of the body to infection; platelets form 5 percent of the total, and they perform a functiuon related to blood clotting.

Shearing Stress -



Two types of forces act on a fluid element. One of them is body force and the other is surface force. The body force is proportional to the mass of the body on which it acts while the

surface force acts on the boundary of the body and so it is proportional to the surface area.

Suppose F is a surface force acting on an elementary surface area ds at the point P of surface S. Let F_1 and F_2 be resolved parts of F in the directions of tangent and normal at P. The normal force per unit area is called normal stress and is also called pressure. The tangential force per unit area is called shearing stress. Hence F_1 is a kind of shearing stress and F_2 is a normal stress.

Viscosity -

Viscosity is that property of real fluid as a result of which they offer some resistance to shearing, i.e. sliding moment of one particle past or near another particle. Viscocity is also known as internal friction of fluid. All known fluids have this property in varying degrees. Viscosity of Glycerine and oil is large in comparision to viscosity of water or gases.

1.5 Shear Tensor and Strain Rate Tensor-

The shearing stress between any thin sheets of a fluid is defined as

$$\tau = \frac{Force}{Area}$$

now $$\tau \alpha \frac{U}{y_0}$$

Where y_0 is the distance between two planes s.t. one is at rest while the other is moving with uniform velocity U parallel to itself.

The fluid lies in between these two plates.

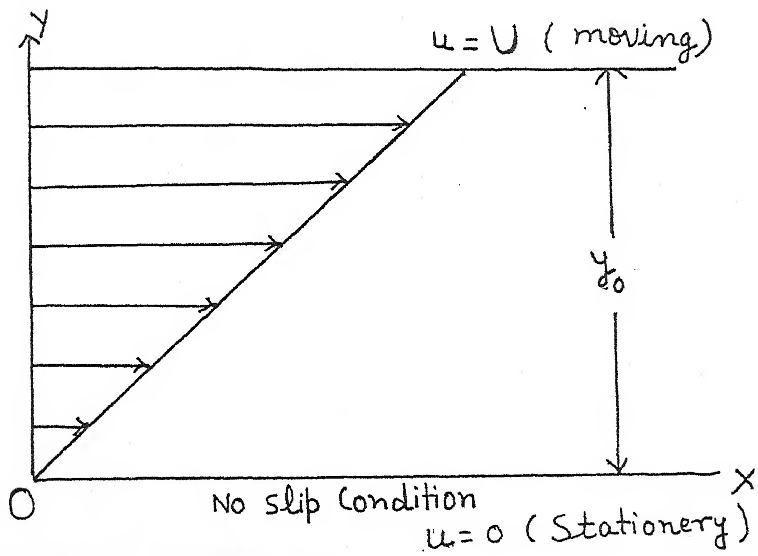
Hence $\tau = \mu \frac{u}{y_0}$

Where μ is the constant of proportionality and is defined as viscocity.

Sometimes $\frac{u}{y_0}$ is denoted by $\frac{du}{dy}$ also it is denoted by $\dot{\tau}$ or by e.

$\frac{du}{dy} = \dot{\tau} = e$ is called velocity gradient,

In the relation $\tau = \mu \frac{du}{dy}$ when μ is constant it is clear that shear stress varies with velocity gradient.



1.6 Non Viscous Fluid -

A fluid is said to be non-viscous fluid if it is incapable of exerting shearing stress. The following have the same meaning. Perfect fluid, Ideal fluid, Inviscous fluid, Non viscous fluid, Frictionless Fluid. In this case $\tau = 0$, so that

$\mu = 0$

1.7 Viscous Fluid -

A fluid is said to be viscous if it exerts shearing stress. Thus in this case

$\tau \neq 0$ so that $\mu \neq 0$

1.8 Elastic Bodies -

Elastic Body is one for which velocity gradient vanishes. In this case

$$\frac{du}{dy} = 0 \text{ so that } \mu = \infty$$

For $\mu = \tau \Big/ \left(\frac{du}{dy}\right)$

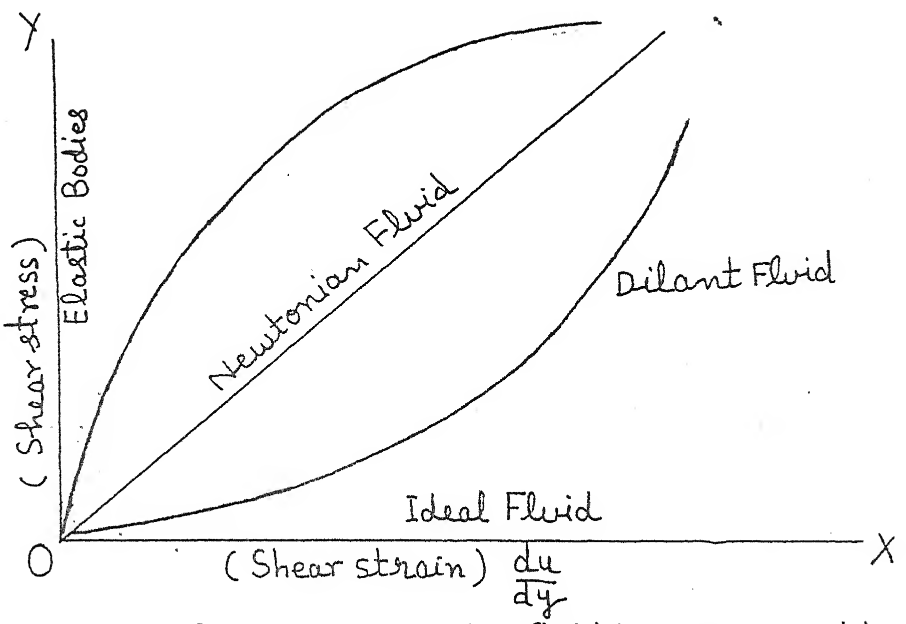
1.9 Newtonian Fluid -

A fluid is said to be Newtonian if its viscosity does not change with the rate of deformation. In this case the equation $\tau = \mu\left(\frac{du}{dy}\right)$ is similar to the equation $y=mx$, where $y=\tau$, $m=\mu$, $x=\frac{du}{dy}$. Hence Newtonian fluid is represented by straight line.

1.10 Non Newtonian Fluid

A fluid is said to be Non-Newtonian if its viscosity varies with the rate of deformation, all are variables in the equation

$$\tau = \mu\left(\frac{du}{dy}\right)$$



Therefore Non-Newtonian fluid is represented by curve. The main classes of Non-Newtonian fluids are Bingham plastics, Pseudo Plastics and Dialants.

or

Non Newtonian fluids are those where stress is Non linearly proportional to strain (velocity gradient)

1.11 Viscocity of Blood -

Blood is neither homogeneous nor Newtonian. Plasma in isolation may be considered Newtonian with a viscosity of about 1.2 times that of water. For whole blood, we can measure effective visocity (apparent viscosity), and this is found to depend on shear rate, The constitutive equations proposed for whole blood are as follows :

i) $\tau = \mu e^n$ (Power law equation)

ii) $\tau = \mu e$ (Newtonian equation)

iii) $\tau = \mu e^n + \tau_0 (\tau \geq \tau_0)$ (hershel - Bulkley equation)

iv) $\tau^{\frac{1}{2}} = \mu^{\frac{1}{2}} e^{\frac{1}{2}} + \tau_0^{\frac{1}{2}}$ (Casson equation)

v) $\tau = \tau y + \mu e$ where μ is newtonian viscosity (Binghan equation)

1.12 Cardiovascular System -

The cardiovascular system consists of the following :

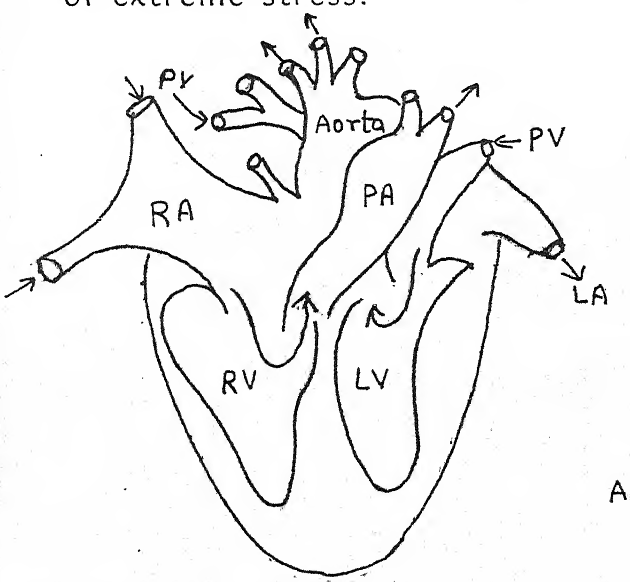
1) The heart (which acts as a pump, whose elastic, muscular walls contract periodically making possible the pulsatile flow of blood.

2) The distributory system (comprising arteries and arterioles for sending blood to the various organs of the body)

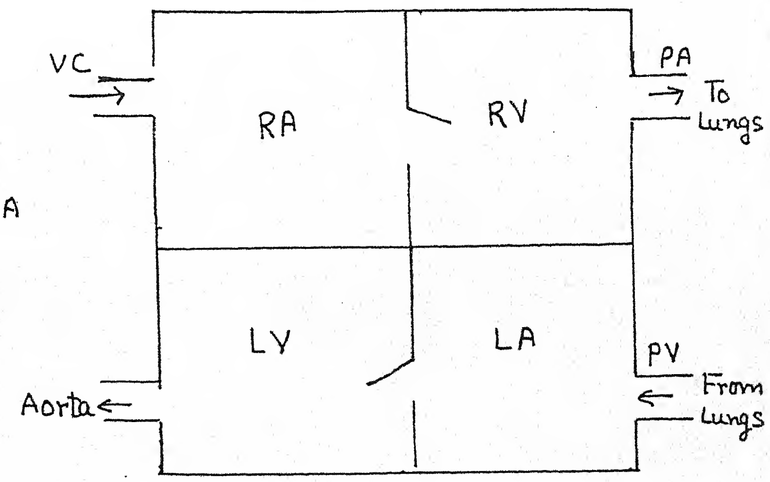
3) The diffusing system (made up of fine capillaries which are in contact with the cells of the body)

4) The collecting system of veins (which collects blood deplected of oxygen and full of products of metabolic processes of the system).

The organs which supplement the function of the cordivascular system are (i) the lungs which provide a region of interphase transfer of O_2 to the blood and removal of CO_2 from it, and (ii) The kidney, liver, and spleen, which help in maintaining the chemical quality of blood under normal conditions and under conditions of extreme stress.



The heart



Deoxygenated blood enters the right atrium (RA) from where it goes to the right ventricle (RV) when the heart contracts, the tricusped valve between the RA and RV closes and blood is pushed out to the lungs through the pulmonary artery (PA) which branches to the right and left lungs where $(O_2$ is removed and blood is oxigenated. The blood returns from the lungs through the pulmonary vein (PV) to the left atrium (LA) and then it goes to the left ventricle (LV) and from there, due to contraction of the heart, it enters the arota from which it travels to other arteries and the rest of the vascular system.

1.13 Fahraous Lindqvist Effect (F-L effect) -

In arteries, blood flows in two layers, a plasma layer near the walls consisting of only the plasma and almost no cells and a core layer consisting of red cells in plasma.

1.14 Blood as a Transport Medium for the Body -

Blood, the red fluid of the blood vessels with which we are all familiar, is the transport medium of the body. It is the medium by which all living tissues are related to their external environment; i.e. from the outside world. Blood is pumped to and fro between tissues and elementary canal to take up oxygen and nourishment respectively, and it helps in returning the products of oxidation and metabolism in the tissues to the outside world by the lungs, the skin and kidney. It is also the medium by which growth and repair substances are transported to the tissues and by which the various controlling glands of the body can distribute their chemical messengers.

Blood is some what viscid fluid. In man and in all other vertebrateanimals with the exception of two (the amphioxus and leptocephalus), it is red in colour. It consists of a continuous yellowish aqueous phase, the plasma, in which formed elements are suspended. The formed elements consists of red blood cells (erythrocytes), white blood cells (leukocytes) and Platelets (thrombocytes). Plasma is made up of water (92%) and contains traces of inorganic and organic salts. The inorganic and small organic molecules contribute little to plasma viscosity which is primarily dependent on the plasma proteins. If blood is allowed to clot and the solid material is removed, the remaining fluid is called serum. This has essentially the same composition as plasma except that fibrinogen and some of the clotting factors have been removed. About 7% by weight, of plasma is protein mainly albumin, globulin and fibrinogen; which have molecular weight ranging from 44,000 to 10,00000. About half of the protein mass is albumin. the significance of plasma protein is its multi natures mentioned below :

1- Plasma Proteins are responsible for osmotic pressure, the level of which is important for regulating water exchange between blood and tissues.
2- They posses buffer properties and maintain the acid-base equilibrium of blood.
3- They produce a definite viscosity of plasma which is important in maintaining blood pressure.
4- They promote stabilization of the blood by providing conditions that prevent sedimentation of crythrocytes.

5- They play an important role in coagulation.

6- Plasma proteins are important factors in immunity.

If blood to which an anticoagulant has been added, is poured into a rest tube and centrifuged, the corpuscles tend to settle at the bottom and the blood is divided into two layers, namely, a red lower layer consisting of the formed elements and a transparent colour-less or slightly yellowish upper layer consiting of plasma. The Leukocytes form of this white film between the crythrocytes and the plasma since their specific gravity is less than that of the erythrocytes.

The red cells surface carries a net negative charge but in stationary blood, cells interact with each other to form aggregates. The aggregates commonly consist of 6 to 10 cells stocked face to face and such a cluster of cells in called a rouleaux. Secondary aggregation of rouleaux also accurs building up a complex three dimennsional structure. When blood is sheared, these rouleaux break up and at sufficiently high shear rate the cell exists as an individual. Blood flows in the form of different laminar containing different types of cells. Considering these factors, the Rheological properties of blood might be expected to be rather complex.

In normal blood cellular components others than red cells compose only about 3% of the total cell volume and usually have a small effect on blood viscosity. White cells may significantly inscrease blood viscosity in deseases where there is large increase in Leukocytes concentration.

The red blood cells are about 97% of the total cell volume in the blood. Consequently, the removal of white blood cells and

platelets does not measurably modify the experimentally determined flow properties of these suspensions. The concentration of most red blood cells in suspension is generally reported as the suspension volume fraction occupied by the red blood cells, called hematocrit.

The hematocrit is normally about 42-45% of volume. The erythroytes consist of a thin flexible unstrechable membrane with an interior filled with saturated hemoglobin solution (Viscosity 6.0 centipose) The membrane is highly deformable if the change in the surface area is small, but becomes very much stiffer if the deformation produces a large change in the area of the cell membrane.

The red blood cells have the shape of biconcave disc which can deform, however into a bullet shaped entity during passage through small capillaries. The cell has a large surface area relative to its volume.

Two properties of blood are essential for the preservation of life. The first is that blood remains fluid in the blood vessels throughout its life and second is that it rapidly becomes solid when shed. The maintenance of fluidity is necessary for the circulation of blood, while the solidification of the shed blood provides indispensable defence against excessive bleeding from wounds. The coagulation of the blood is due to formation of a jelly by the deposition of protein material called fibrin and it is the formation of this body that the fundamental change in blood clotting occurs.

To prevent this from occuring, various anticoagulents are added to the blood when it is drawn from the animal.

The effects of an anticoagulent on rheological properties of

blood are difficult to assertain because of the difficulty in making reliable rheological measurements in the short time available between the blood drawing and the blood clotting. The effect of anti coagulents on blood viscosity appears to be small.

1.15 Human Circulatory System -

In the human body circulatory system of blood consists of four functions Respiratory, Nutritive exetretory, Protective and Regulatory. The heart that provides the energy for the circulation has four compartments (ie left and right atrium and left and right ventricles) interconected to each other by one way valves. Blood coming from body tissues enters the right atrium. Through the venul caval contraction of right atrium forces blood pass the tricuspid valves into the right ventricle. From this point there begin two subdivisions the pulmonary and the systematic circulation. As the term shows, the former services the lung and latter the various systems of the body. The blood vessel have internal diameter in the range of 2.5 cm in aorta to about 4 microns in capillary.

Blood is pumped from the heart into the aorta from where it goes to the circulatory system consisting of about 40 large arteries, 1600 main artery branches, 1800 terminal branches, 4,00,00,000 arterioles, 1,20,00,00,000 capillaries, 8,00,00,000 valves, 1,800 terminal veins, 1600 main venous branches, 40 large venous branches, 40 large veins and then returns to the heart through the venule caval.

It is estimated that the heart muscles it self consume about 18% of the energy required to sustain life.only some energy

gose in to the mechanical work of pumping blood. The heart beats about 70 times in one minute in an average person at rest. Blood flow in smaller blood vessels approaches steady flow condition otherwise pulsatile. The pressure in the aorta rises rapidly to its maximum (systolic) value of about 120 mm/Hg. The requirement of the circulation is the supply of oxygen required for metabolic process. In an average man at rest, the O_2 required is 200 ml/ minute for under physical stresses, the need may rise to 5 litre/min. In an average man it is necessary for the heart to circulate 5 to 6 litres of blood per minute or about the 5 times the metabole requirement of oxygen. The metablic requirements are the energy requirements for biological functions.

In fluid mechanics the circulatory system is classified in to two parts viz. macrocirculation and microcirculation.

1.16 Macrocirculation -

The flow of blood in vessels of diameters greater than 500 μ m (such flow occcures in aorta) is called macrocirculation. The flow is characterized by high Reynolds number, defined as the ratio of inertial to viscous forces. Turbulent flow can occur in blood vessels for the Reynolds number greater than 2300. The governing equations for analysing of such flow conditions should include inertial effects, effect of curvature of blood vessels, pulsatile flow and the distensiblity of the vessel wall. Blood can be taken as homogeneous and continuous fluid when vessel diameter is large in comparison to the dimensions of red blood cells. Pulsatile flow effects and Pressure wave propagations have been studied in macrocirculation by a number of research workers, Skalek and Taylor. The flow disturbances at bifurcations, bends etc. and

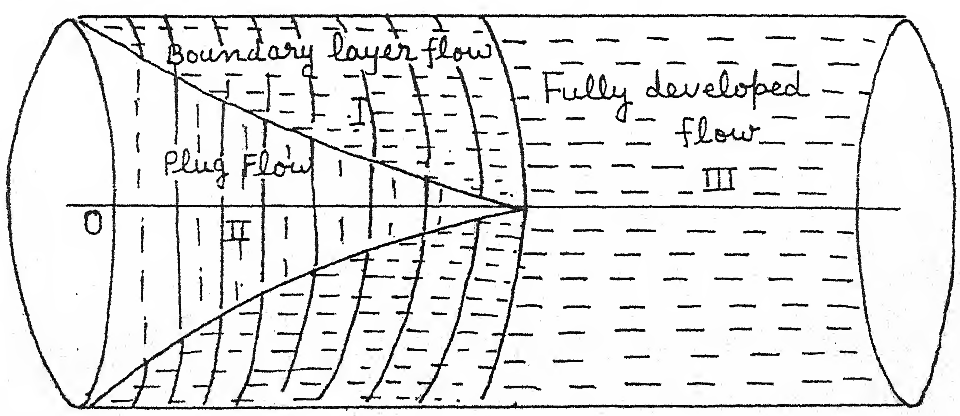
their effects on Pathological states are studied by Patel.

1.17 Microcirculation -

When the diameter of blood vessel is less than 500 μ m [i.e arterioles, capillaries, venules] the circulation is called microcirculatory.This is responsible for 80% pressure drop in circulatory system. In capillary bed the transfer of nutrients to and removal of wastes from the living cells of the body is a part of microcirculation. The flow is characterized by verylow Reynolds numbers (Re<1). But we can not neglect the size of the red blood cell, compared to blood vessels size.

1.18 Boundary Layer Flow, The Cone Flow and Fully Developed Flow of Fluid -

When a fluid enters a tube from a large reservoir where the velocity is uniform and parallel to the axis of the tube, the velocity profile is a flat surface at the entry.



Immediately after entry, the velocity near the surface is affected by the friction of the surface, but the velocity profile near the axis still remain flat. As the fluid moves further in the tube, the flat portions decreases, and at the section corresponding to A, the paraboloidal velocity profile for the fully developed flow is reached. The flow in the region OA is called the entry region (or inlet) flow and the flow beyond A (in region III) is called the fully developed flow. The length OA is called the entry length. The flow in the entry length portion it self consistsof two parts. The flow in region I near the surface is called the boundary layer flow, the flow in region II is called the core flow or the pluge flow.

In plug region, $e = \dot{\gamma} = \frac{dv}{dr} = 0$

& $\tau \leq \tau y$ where τy is yield stress

& v = constant

ie plug flow exists where ever the shear stress does not exceed the yield stress.

1.19 Rheology of Blood -

Blood behave like a time dependent non Newtonian fluid and the basic rheological property of blood is its viscosity. the viscosity of whole blood is about 4.0 centipose and of plasma is about 1-2 centipose at 37°C. The specific gravity of whole blood is 1.05 to 1.06 and that of plasma is about 1.03. Thus the red blood cells tend to sediment slowly in plasma. The increased viscosity relative to water is produced mainly by the presence of plasma proteens. Both, the molecular shape and concentration of the protein are important. Fibrinogens which has an elongated molecule

has a marked influence; although forming less than 5% of the total plasma proteins. . It is responsible for about 20% of the plasma viscosity elevations.

1.20 The Viscosity of Blood -

It is an important factor in determining the local pressure variation through the cardio-vascular system which in turn influences the localflow rates through each section of the vascular network. The clinical importance of blood viscosity as a parameter lies in its sensitivity to small variations in composition. One can aften diagnose pathological states by determining a change in blood viscosity.

There are several rheological parameters (e.g plasma, blood cells, hematocrit etc.) which can effect the blood viscosity. the viscosity of plasma increases with its protein concentration. But some proteins have different influences on plasma viscosity depending on their shape and size. The influence of Fibrinogen on plasma viscosity can be seen in the difference between plasma and serum viscosities. Serum usually has a viscosity which is 20% less than that of plasma. Many relationships have been suggested to express blood viscosity as a function of cell concentration, plasma viscosity and shear rate. When temperature is increased, the viscosity of blood and plasma is fallen. Measurements should be made at constant tamperature (37°C).

Platelets and white cells, in general have little influences on blood viscosity, because they are present in very minute quantity as compared to red cells.

. The deformation of red blood cells alloow the blood to remain fluid up to hematocrit of 98%; rigid cell without deformation will cease to flow at a rigid cell without deformation will cease to flow at a cell concentration of about 60%. The hematoerit also influence the deformation of the erythrocytes i.e raising of the cell concentration produces an increase in cell deformation and therefore fall in viscosity of blood occurs.

In stationary blood, rouleax is formed which intracts to produce larger aggregates. At low flow rates the presence of these red cell structures strongly influence the viscosity of the blood. The size of the rouleaux and aggregates progressively decreases as the. shear rate increases. In normal blood the disaggregation is probably complete at shear rate 50 $sec.^{-1}$ approximately. The viscosity of blood increases when shear rate falls, but it is unceration as to what happens when the shear rate actully falls to zero. The build up of a 3-D of increasing aggregates suggests that blood may show a yield stress.

1.21 Blood as a Non-Newtonian Fluid -

Blood, from fluid mechanics point of view can be thought as a suspension of erythrocytes in a Newtonian liquid called plasma. In small vessels the dimensions of red blood cells are not negligible as compared to the size of tube. Therefore blood can not be considered as a homogeneous fluid in microcirculatory systesm (diameter less than 500 m,) It is a specific property of blood that exhibits a yield stress. Hence if applied shear stress is below a critical value ($\tau < \tau y$) the response will be elastic and on removal of stress the shape of blood film is unaltered. othesrwise ($\tau > \tau y$)

flow takes place and blood behaves as a non Newtonian fluid.

It is found that suspended blood cells are responsible for the non Newtonian nature of blood rheology. At low shear rates the blood exhibits yield stress and behaves like casson fluid $[\tau^{\frac{1}{2}} = \tau y^{\frac{1}{2}} + \mu^{\frac{1}{2}} e^{\frac{1}{2}}]$ constitutive equations suggested by Herchel and Bulkley $(\tau = \tau y + \mu e)$ has also been used to describe the shear rate dependence of blood.

Blood rheology can be helpful in diagonosis of some blood problems.

1.22 Survey of literature -

The German form of the word (rheologic) and the description of small viscometer as a microrheometer are found in literature since early days. But in its modern sense, the term rhchology was coined by prof. E.C. Bingilam and formally adopted and defined at the foundation meeting of the American society of rheology in December 1929 in Washington, as " The science of deformation and flow of matter"

However, the subject of hydrodynamics andaerodynamics are not included in rheologhy.

The first mathematical paper of blood flow was given by Leonhard Euler in 1862. He developed one diamensional equations of invisicid flow of an incompressible fluid in an elasitic tube. Blood flow in capillaries is of great interest to Phisiologists involved in microvascular research. The pressure drop relation in microcirculation was oobtained by Posiseuille in 1940. The most remarkable result on blood flow was of Fahraeus and Lindguist,showing that the apporent viscosity (effective viscosity)

of blood decreases as tube diameter decreases from 500 μm. This effect has been confirmed by other investigators (Coklet and Dintenfass)0. Taylor studied dispersion process in Newtonian fluid flows- through a circular tube.

1.23 Geometrical aspect of Vessels -

A striking characteristic of the circulatory system is its geometrical complexity. Blood must flow through many bends, bifurcations, stenosis and tapering during its journey, Most of the progress in understanding the mechanics of the circulations is based on the investegations of the flow in straight uniform tubes .

It is only within the last two decades that the effects of geometric transitions on blood flow have begun to be explained.

1.24 Stenosis -

Stennosis causes the narrowing of the blood vessels due to the development of abnormal tissues and gives way to serious circulatory disorderby reducing the blood supply. Hemodynamic charaterestics may be changed due to this undesirable growth which could be injurious to normal health. In recent years many workers have investigated the flow charcacteristies of blood through artery in presence of mild stenosis.

Young gave a theoritical analysis of the effect of time dependent stenosis on flow characteristises of blood. Recently Sinha and Singh studied the effect of stenosis on blood flow through the couple stress..

1.25 Tapering -

The mammalian arterial consists of the main aortic tube which continues into the artelaries and a number of side branches. Transformations and propagations of pressure and flow waves depend on the destribution of the characteristic impedence. The two contributing factors are the decrease of the cross sectional area and she peripheral increase of the valve stiffness. Many blood vessels have the characteristic of taper cylinders rather than straight cylinders. Block gave the idea that vessels which carry blood towards the tissues should be considered as long, slowly tapering cones rather than cylinders. The relations for shear stress and their variations with suspension concentration and tapered angle, have been discussed in this thesis.

Table I
Basic information

Vessel	Diameter (cm)	Length (cm)	Wall Thickness	Average Vescocity (cm/see)	Average Reynolds Number	Maximum Velocity (cm/see)	Maximum Reynolds Number
Arota	2.5	50	0.2	48	3400	120	8500
Arteries	0.40	50	0.1	45	500	90	1000
Arterioles	0.005	1	0.02	5	0.7		
Capillaries	0.0008	0.1	0.0001	0.1	0.002		
Venules	0.002	0.2	0.0002	0.2	0.01		
Veins	0.5	2.5	0.05	1.0	140		
Venacava	3.0	50	0.15	38	3300		

Table 2

Mean Pressure (mm Hg)in Blood Vessels

Arota	Arteries	Arterioles	Capillaries	Venuls	Veins	VenaCava	RA	RV
100	90 →	75 →	45 →	25 →	10 →	5 →	5 →	25
⇑								⇓
LV	LA	Veins	Capillaries	Arterioles	Arteries	Pulmonary artery		
100 ←	5 ←	5 ←	10 ←	15 ←	20 ←	25		

Table 3
Protein Concentration in human Plasma

	Conc. (g/Litre)	Molecular Weight (ml.)	Mol.Sizes (μm)
Protein	75		
Albumin	45	69,000	15 x 4
Globubin	27	1,50,000	24 x 4.5
Fibrinogen	3	3,40,000	70 x 4

Chapter II

Rheological Behaviour of Steady Blood Flow in Narrow Vessels

2.1 Introduction.

From phisiological aspect blood flow throw narrow circular tubes is of great interest in microcirculatory system. From mathematical point of view blood is considered as a suspension of erythrocytes (red blood cells) in a Newtonian fluid called plasma when the diameter of red blood cells are no longer negligible as compared to the tube size. The apparent (effective) viscocity of blood depends on many factors such as, plasma viscocity, hematocrit, Size of vessel, shear rate, velocity of flow, rigidity etc. Further, the apparent bloodviscocity in capillaries is much lower than that in large vessels (Dintenfass-) It is a well Known fact that apparent viscocity of blood decreases as the tube radius decreases . This

is known as the Fahraeus Lindguist effect. Many workers in the field have shown that the fluid dynamic properties of blood flow like velocity profile, velocity gradient, shear stress at the wall, apparent viscocity of blood, deformability of red cells could play a vital role in the. fundamental understanding, diagnosis and treatment of many deseases (cardio- vascular, diabetis etc). Many mathematical models have been provided for blood by a number of research workers,e.g. P.Chaturani, Nand Lal Singh, Charm et. al.

Bugliarello and Sevilla have proposed two fluid models, in which either of both layers (Peripheral plasma layer and core hematocrit layer)l are of Newtonian fluid with different stress and vicosities.

It has been found that for blood flowing in narrow tubes(500μm- 20 μm), a cell free plasma layer occurs near the wall, whose diamension effects the blood behaviour to a large extent.

Scott Blaier and Spanner reported that in normal range of shear rates (except at very high or a very low shear rate). There is no difference between Casson plots and Herschel Bulkley plots of experimental data and in some cases latter model provides better results.

Many authors have the view that the slip at the solid surface has no evidence. Therefore we have neglected the slip velocity at the wall in this chapter.

In our analysis we have considered the two layer flow model, constitutive equations of motion and the expression for velocity fields, flow rate and variation of flux with τ_0 obtained.

2.2 Mathematical Analysis -

In the present analysis we have considered as incompressible viscousfluid model of steady blood flow throw narrow vessels under a uniform pressure gradient. It is assumed that the radius of vessel is constant.

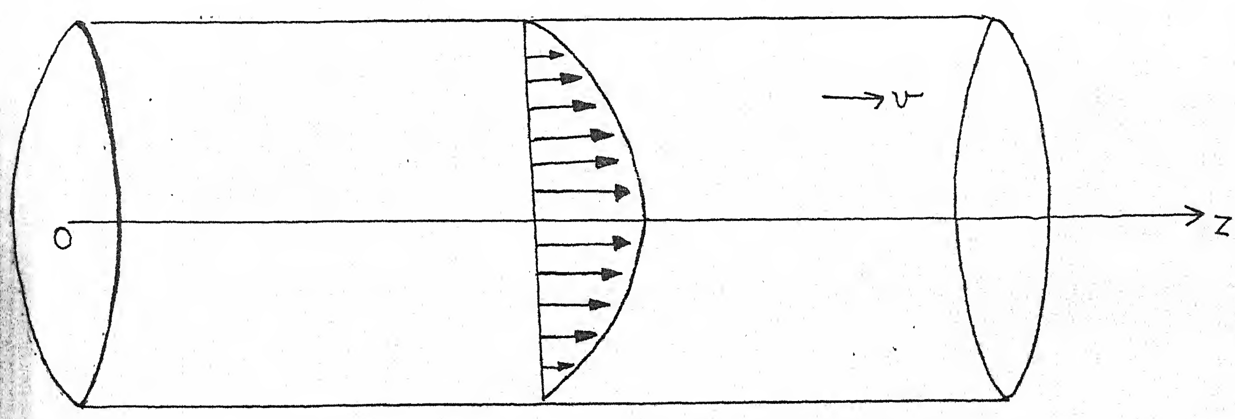
Governing equations in cylindrical polar cordinate system (r, θ, z) where the axis of symmetry is taken as the axis of Z. There are, in general, three components of velocity, namely, Vr along the radius vector perpendicular to the axis, Vo perpendicular to the axis and the radius vertor, and Vz parrallel to the axis of Z, for the axe-symmetric case, we take Vo=0 and we also take Vr, Vz and p to be independent of θ. In this case the equation of continuity and the equations of motion are given by

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{\partial}{\partial z}(v_z)=0 \qquad (1)$$

$$\rho\left(\frac{\partial vr}{\partial t}+v_r.\frac{\partial vr}{\partial r}+v_z\frac{\partial vr}{\partial z}\right)=-\frac{\partial P}{\partial r}+\mu\left(\frac{\partial^2 vr}{\partial r^2}+\frac{\partial^2 vr}{\partial z^2}+\frac{1}{r}\frac{\partial vr}{\partial r}\right)-\frac{vr}{r^2} \qquad (2)$$

$$\rho\left(\frac{\partial vz}{\partial t}+v_r\frac{\partial vz}{\partial r}+v_z\frac{\partial vz}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 vz}{\partial r^2}+\frac{\partial^2 vz}{\partial z^2}+\frac{1}{r}.\frac{\partial vz}{\partial r}\right) \qquad (3)$$

Hagen Poiseuille Flow :



Velocity Profile for Poiseville Flow

We consider steady flow when there is only one velocity component parallel to the axis so that Vr=o, $V_\theta=0$ and Vz=v.

Then the equation of continuity gives

$$v_z = v(r) = v \qquad (4)$$

The equations of motion (2) and (3) give

$$\frac{\partial p}{\partial r} = 0 \quad \& \quad \frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} = \frac{1}{\mu}\frac{\partial p}{\partial z} \qquad (5)$$

From equation (5), $\left(\frac{-\partial p}{\partial z}\right)$ must be a constant. Let us denote this constant pressure gradient by P. then

$$\frac{1}{r}\frac{1}{dr}\left(r\frac{dv}{dr}\right) = \frac{-P}{\mu} \qquad (6)$$

Integrating this equation

$$r\frac{dv}{dr} = \frac{-Pr^2}{2\mu} + A$$

$$v(r) = \frac{-Pr^2}{4\mu} + A\log r + B \qquad (7)$$

But velocity on the axis (i.e. at r=0) must be finite

$\therefore$ A=0, Then at r = a, v = o

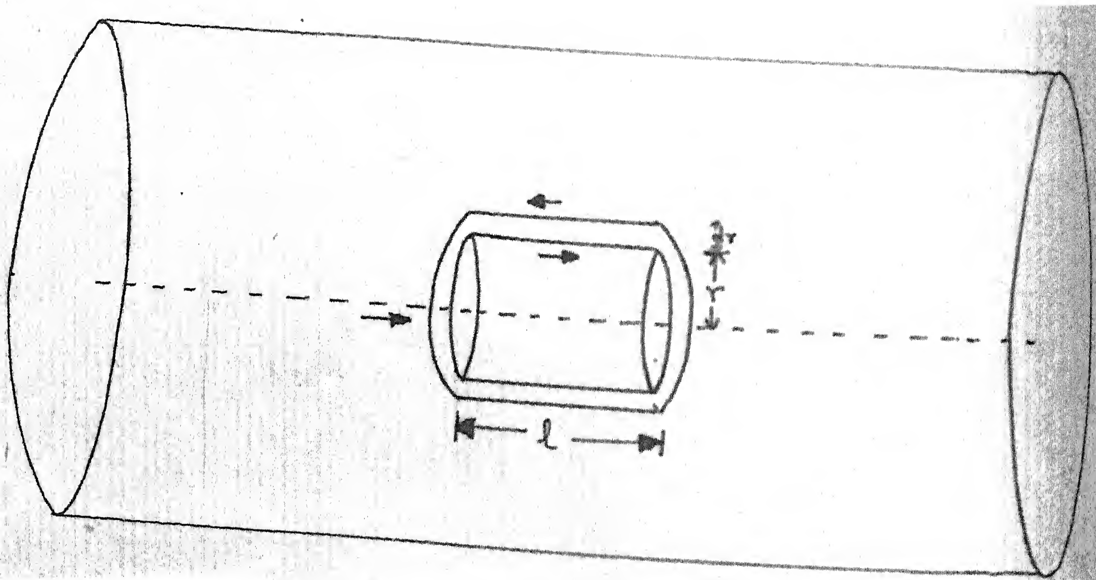
$$\because 0 = \frac{-pa^2}{4\mu} + 0 + B$$

$$v(r) = \frac{-Pr^2}{4\mu} + \frac{Pa^2}{4\mu} = \frac{P}{4\mu}\left(a^2 - r^2\right)$$

$$\therefore v = \frac{P}{4\mu}\left(a^2 - r^2\right) \qquad (8)$$

The total flow across any section i.e. total volume of fluid crossing any section per unit time is given by

$$Q = \int_0^a v.2\pi r \cdot dr$$



Forces on Control Volume

$$= \frac{\pi a^4 P}{8\mu} \quad \text{(on integration)}$$

$$\therefore Q = \frac{\pi a^4 P}{8\mu} \tag{9}$$

2.3 Steady Non-Newtonian Fluid Flows in Circular Tubes

Basic Equations for Fluid Flow

We consider the laminar flow of a non-newtonian fluid in a circular tube under a constant pressure gradient. Let the control volume be bounded by two coaxial cylinders of radii r and r+dr and let it be of unit length. Due to pressure gradient, there is a forward force px2 πr dr on it. Let the stress be $\tau(r)$ at a distance r from the axis. Then the force on the inner cylindrical surface is $2\pi r\tau$, and the force on the outer cylindrical surface is

$$2\pi(r\tau) + \frac{2\pi d}{dr}(r\tau).dr$$

Balancing the forces in the axial direction on the control volume, we get

$$2\pi \frac{d}{dr}(r\tau) = 2\pi r P$$

$$\text{or} \quad \frac{d}{dr}(r\tau) = \text{Pr} \tag{1}$$

Integrating (1), we obtain

$$r\tau = \frac{1}{2}Pr^2 + A \quad \text{or} \quad \tau = \frac{1}{2}P\left(r + \frac{D}{r}\right) \tag{2}$$

Since the stress is finite on the axis (i.e. at r=0) We have

$$A=o,\ D=0,\ \tau = \frac{1}{2}\text{Pr} \tag{3}$$

The velocity v is parallel to the axis. It is a function of r

only and is expected to decrease from a maximum on the axis to zero on the surface so that the only non zero component of strain rate is

$$e = -\left(\frac{dv}{dr}\right) \tag{4}$$

For a non-Newtonian fluid

$$\tau = f(e) \tag{5}$$

From (3) - (5)

$$\frac{1}{2}\mathrm{Pr} = f\left(-\frac{dv}{dr}\right) \tag{6}$$

Integrating (6) subject to the condition that v=o when r=R, we get v as a function of r, Then we can obtain the flux Q by using

$$Q = \int_0^R 2\pi r v dv \tag{7}$$

Integrating right-hand side of (7) by parts we get

$$Q = 2\pi\left[\left(\frac{1}{2}r^2 v\right)_0^R - \int_0^R \frac{1}{2}r^2\frac{dv}{dr}dr\right] \tag{8}$$

Since v=o at r=R we get

$$Q = \pi\int_0^R r^2 e(r)dr \tag{9}$$

2.4 Flow of Power-law Fluid in Circular Tube

Here $\tau = \mu e^n$

$$\frac{dv}{dr} = -\left(\frac{1}{2}\frac{\mathrm{Pr}}{\mu}\right)^{\frac{1}{n}} \tag{10}$$

Integrating (10) we obtain

$$v = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right) \tag{11}$$

Also

$$Q = \int_0^R 2\pi rvdr = \left(\frac{1}{2}\frac{P}{\mu}\right)^{\frac{1}{n}} \frac{n\pi}{3n+1} R^{\frac{1}{n}+3} \quad (12)$$

2.5 Flow of Herschel - Bulkley Fluid in Circular Tube

In this case, we have e=0 when $\tau \le \tau_0$. and there is a core region which flows as a plug. Let the radius of this plug region be rp. At the surface of the plug, the stress is τ_0 so that, considering the forces on the plug, we get

$$P x \pi rp^2 = \tau_0 x 2\pi rp$$

$$\text{or } rp = \frac{2\tau_0}{p} \quad (1)$$

In the non-core region, $\tau \ge \tau_0$ and

$$\tau = \mu e^n + \tau_0 \quad (2)$$

$$e = \left(\frac{\tau - \tau_0}{\mu}\right)^{\frac{1}{n}} = -\frac{dv}{dr} \quad (3)$$

$$\frac{dv}{dr} = -\left(\frac{1}{2}\frac{p}{\mu}\right)^{\frac{1}{n}} (r - rp)^{\frac{1}{n}} \quad (4)$$

Integrating (4) we obtain

$$v = \frac{n}{n+1}\left(\frac{p}{2\mu}\right)^{\frac{1}{n}} \left[(R - rp)^{\frac{1}{n}+1} - (r - rp)^{\frac{1}{n}+1}\right] \quad (5)$$

If r=rp then v=vp (the velocity of the plug flow) so that

$$vp = \frac{n}{(n+1)}\left(\frac{p}{2\mu}\right)^{\frac{1}{n}} (R - rp)^{\frac{1}{n}+1} \quad (6)$$

Egu (1) determines the radius of the plug and then using this value of the plug, (6) determines the velocity of the plug and (5) determines the velocity in the non-core region.

Also

$$Q = \pi r p^2 v p + \int_{rp}^{R} 2\pi r v dr$$

$$= \pi r p^2 \frac{n}{(n+1)}\left(\frac{p}{2\mu}\right)^{\frac{1}{n}}(R-rp)^{\frac{1}{n}+1} +$$

$$\frac{n}{(n+1)}\left(\frac{p}{2\mu}\right)2\pi\left[\frac{1}{2}(R-rp)^{\frac{1}{n}+2}(R+rp)-\frac{(R-rp)\frac{1}{n}+3}{\frac{1}{n}+3}-\frac{rp(R-rp)^{\frac{1}{n}+2}}{\frac{1}{n}+3}\right]$$

$$= \pi\frac{n}{(n+1)}\left(\frac{P}{2\mu}\right)^{\frac{1}{n}} R^{\frac{1}{n}+3}$$

$$\left[C_p^2(1-C_p)^{\frac{1}{n}+1}+(1+C_p)(1-C_p)^{\frac{1}{n}+2} - \frac{2}{\frac{1}{n}+3}(1-C_p)^{\frac{1}{n}+3}-\frac{2C_p}{\frac{1}{n}+2}(1-C_p)^{\frac{1}{n}+2}\right] \qquad (7)$$

$$= \pi\frac{n}{(3n+1)}\left(\frac{P}{2\mu}\right)^{\frac{1}{n}} R^{\frac{1}{n}+3} f(Cp) \quad \text{(say)} \qquad (8)$$

Where $C_p = \frac{r_p}{R} = \frac{2\tau_0}{PR}, f(0) = 1$

If Q_0 denote the flux when there is no plug flow (i.e where $\tau = 0$, Cp=0) we get

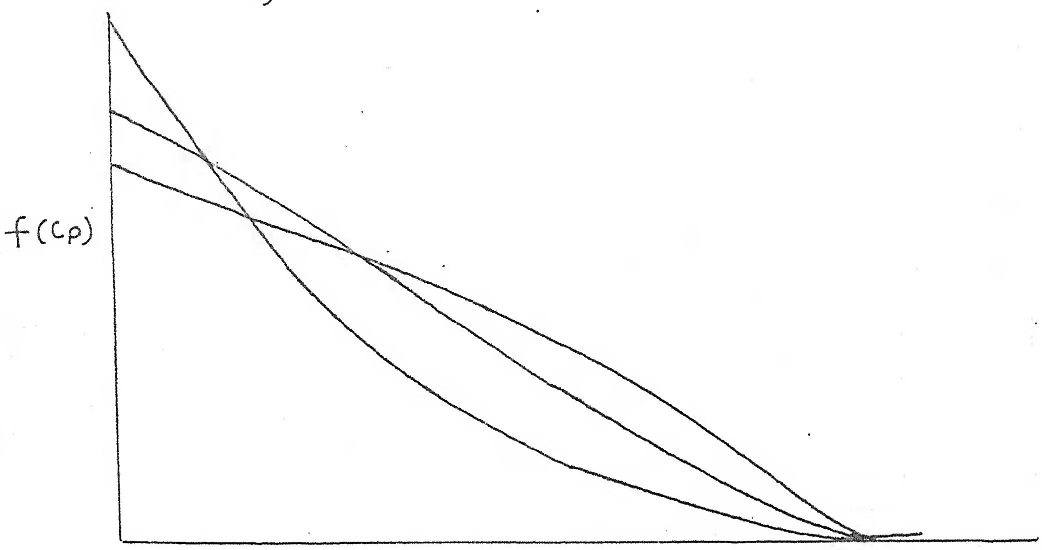
$$\frac{Q}{Q_0} = f(C_p) = f\left(\frac{r_p}{R}\right)$$

$$= f\left(\frac{2\tau_0}{PR}\right)$$

Results & Discussions -

This gives the relative change in Q with τ_0, Figure.2.1 illustrates the vaiations of $f(C_p)$ with C_p for various values of n, The figures shows that

(i) As τ_0 increases (μ and n remaining the same), the flux decreases rapidly and approaches zero as cp approaching unity.



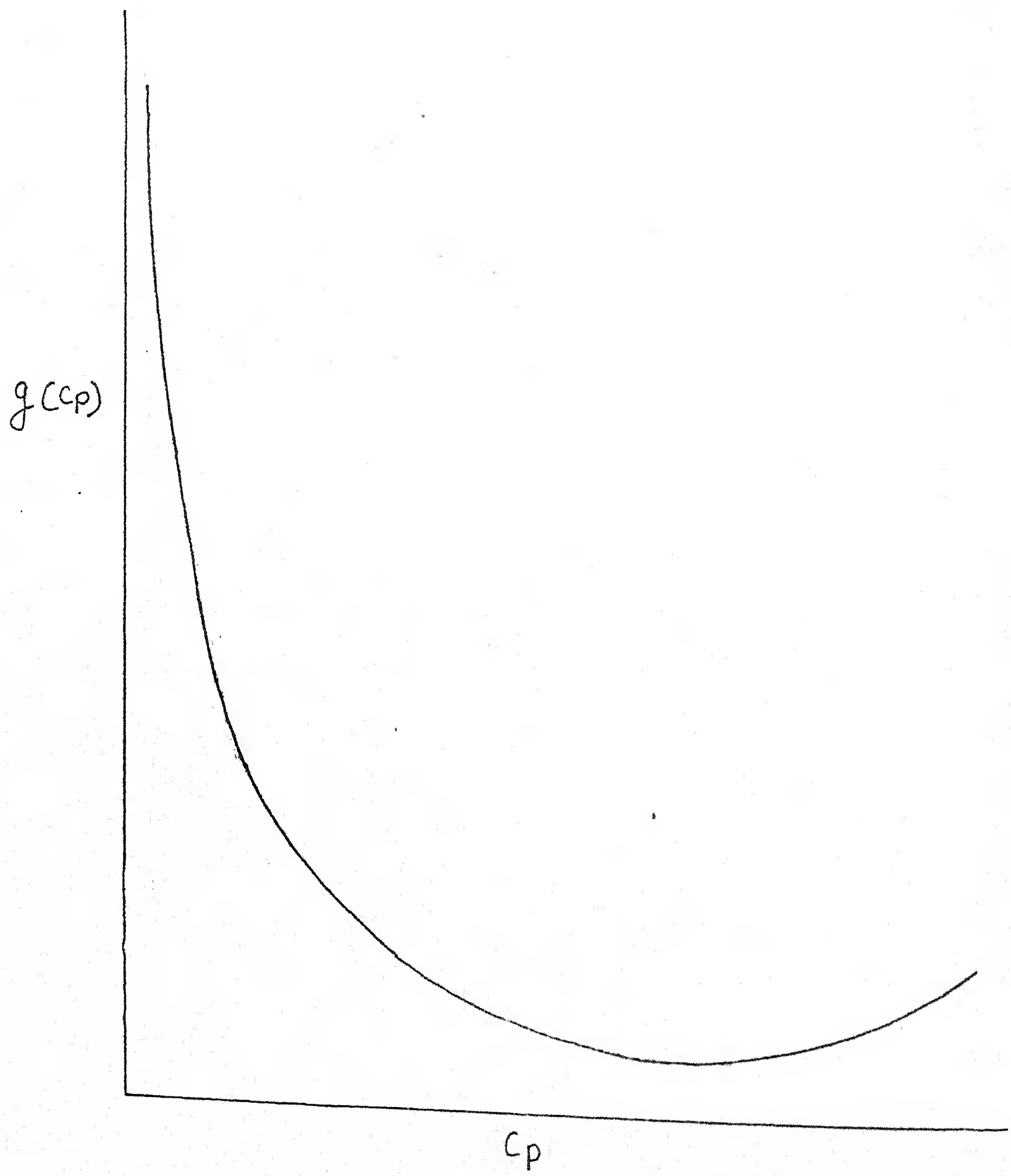
Variation of flux with τ_0

Fig 2.1

(ii) If n < 1, the curve is always concave upwards; when n=1, the curve is always a straight line in the beginging and becomes concave upwards's and when n>1, the curve is convex in the begining and becomes concave near C_p=1 and therefore, it has a point of inflexion

(iii) If τ_0 and μ are constant, the decline in Q is more when n<1 and is less when n>1. If we put n=1 in (7) & (8) we get the results for the special case of a Bingham Plastic.

If we put $\tau_0 = 0$, $\gamma_p = 0$ in (7) we get results for the special case of a power law fluid. Further if we put n=1, we get results for Poiseville flow.



Variation of g(Cp) with Cp

Fig 2.1

2.6 Flow of Casson Fluid in Circular Tube

Here

$$\tau^{\frac{1}{2}} = \mu^{\frac{1}{2}} e^{\frac{1}{2}} + \tau_0^{\frac{1}{2}} (\tau \geq \tau_0) \qquad (1)$$

So that for the noncore region

$$\frac{-dv}{dr} = e = \frac{\left(\frac{1}{2}pr\right)^{\frac{1}{2}} - \left(\frac{1}{2}P r_p\right)^{\frac{1}{2}}}{\mu^{1/2}}$$

or

$$\frac{dv}{dr} = -\frac{P}{\mu}\left(r^{\frac{1}{2}} - rp^{\frac{1}{2}}\right)^2$$

$$= \frac{1}{2}\frac{P}{\mu}\left(2\sqrt{rp\, r} - r - rp\right) \qquad (2)$$

Integrating (2) we obtain

$$v = \frac{1}{2}\frac{p}{\mu}\left(\frac{4}{3}\sqrt{r_p}\, r^{\frac{3}{2}} - \frac{1}{2}r^2 - r_p r - \frac{4}{3}\sqrt{r_p}\, R^{\frac{3}{2}} + \frac{1}{2}R^2\right) + r_p R \qquad (3)$$

so that the plug velocity is given by

$$v_p = \frac{1}{2}\frac{p}{\mu}\left(\frac{1}{2}R^2 + r_p R - \frac{4}{3}\sqrt{r_p}\, R^{\frac{3}{2}} - \frac{1}{6}r_p^{\,2}\right)$$

$$= \frac{1}{4}\frac{PR^2}{\mu}\left(1 + 2c_p - \frac{8}{3}c_p^{\frac{1}{2}} - \frac{1}{6}c_p^{\,2}\right)$$

$$= \frac{PR^2}{4\mu} g(cp)(say)$$

$$\text{Then} \frac{V_p}{(V_p)_0} = g(cp) \qquad (4)$$

Results and Discussions

Fig. 2.2 shows the variation of g (cp) with cp. This shows

that as τ_0 increases (μ remaining the same). The plug velocity or the maximum velocity of flow decreases rapidly till cp reaches .6 when the velocity is reduced to about 6 percent of the value and then it rises slightly. For blood, samll changes in τ_0 lead to significant changes in maximum velocity.

The flux Q is given by

$$Q = \pi v_p r_p^2 + \frac{p\pi}{\mu}\left[\frac{8}{21}r_p\left(R^{\frac{7}{2}} - r_p^{\frac{7}{2}}\right) - \frac{1}{8}\left(R^4 - r_p^4\right)\right.$$

$$-\frac{1}{3}r_p\left(R^3 - r_p^3\right) - \frac{1}{2}r_p R\left(R^2 - r_p^2\right)$$

$$\left.+\frac{1}{4}R^2\left(R^2 - r_p^2\right) + \frac{1}{2}r_p R\left(R^2 - r_p^2\right)\right]$$

$$= \frac{\pi}{4}\frac{PR^4}{\mu}c_p^2 g(c_p) + \pi\frac{PR^4}{\mu}$$

$$X\left[\frac{8}{21}\sqrt{c_p}\left(1 - c_p^{7/2}\right) - \frac{1}{8}\left(1 - c_p^4\right) - \frac{1}{3}c_p\left(1 - c_p^3\right) - \frac{2\sqrt{c_p}}{3}\left(1 - c_p^2\right)\right.$$

$$\left.+\frac{1}{4}\left(1 - c_p^2\right) + \frac{1}{2}c_p\left(1 - c_p^2\right)\right]$$

$$= \frac{\pi PR^4}{8\mu}h(c_p) say$$

$$To That \frac{Q}{Q_0} = h(C_p)$$

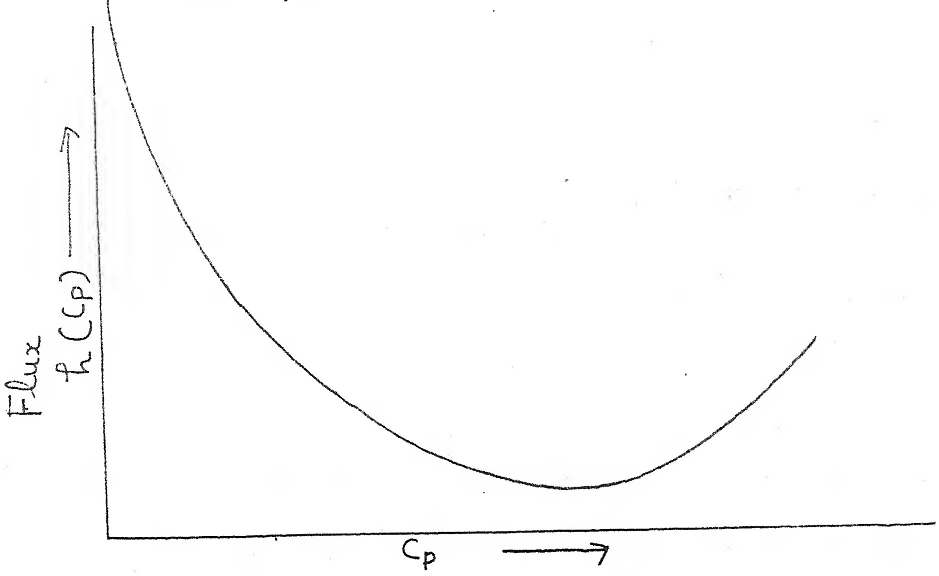
Results and Discussions -

Figure 2.3 gives the graph of h (C_p) against C_p.

It shows that, as τ_0 increases (μ remaining the same), the flux decreases rapidly till C_p= .6 and till it has fallen to about 5

percent of Q_o and then it rises again For blood, small changes in τ_o can make significant changes in Q

The Casson fluid flows in the tube only if $r_p < R$ i.e if $2\tau_o < PR$



Variation of $h(C_p)$ with C_p

Fig 2.3

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Chapter III

A Study of Cason Fluid Model of Blood flowing through a Narrow Tapered Tube

3.1 Introduction -

The behaviour of blood flow is influenced by the suspension of cells in an aguas solution of organic and inorganic substances called plasma. There have been many attempts to explain the anamalous behaviour of blood by proposing defferent theoretical models, e.g. Newtonian and non Newtonian (microplar casson, Herschel- Bulkly, Power lawer etc.)

Iida and Murata (64) have studied pulsatile blood flow through small vessels by assuming Hershel Bulkly fluid model of blood,

Ariman et al. (3), Bugliarello and sevilla, Chaturain and Upadhya and many other workers have investigated the flow of blood through straight circular tubes. Since the vessels bifurcate at frequent intervals, the diameter varies quite rapidly with respect

to distance. Due to the reduction in the vessels diameter at each bifurcation and angle of taper (which is small) thier influence can not be neglected

Block (8) has suggested that all the vessels which carry blood towards the tissues should be considered as long, slowly tapering cones rather than cylinders, Chaturani and prahlad have investigated a steady laminar flow of blood in a uniform tapered tube by considering blood as polar fluid.

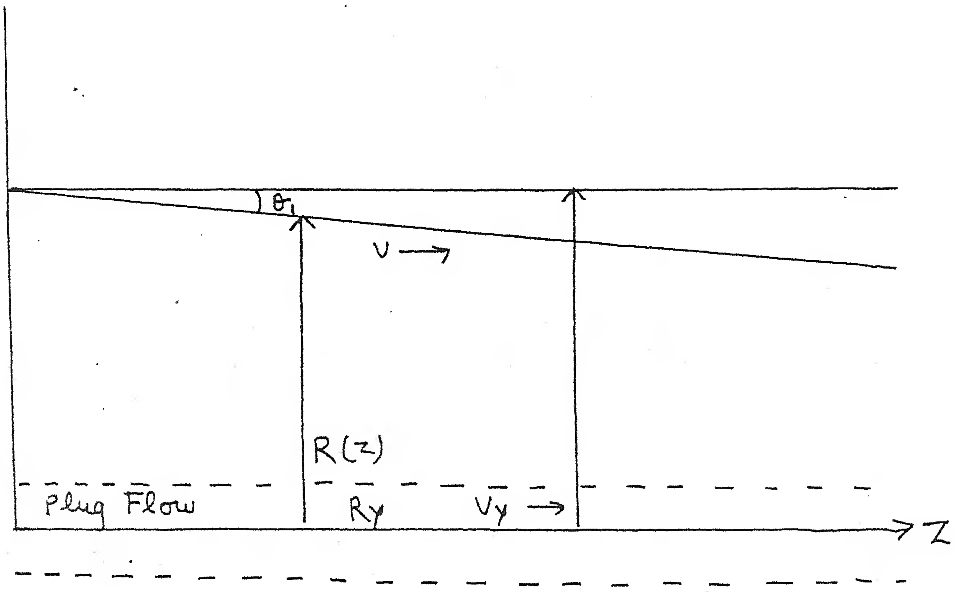
Many research workers like Dwivedi et al have studied steady blood flow in tapered tube by assuming different models of blood.

Some research workers analysed both theoretically and experimentally the aspects of blood rheology in tapered tubes and obtained pressure flow relation

Block and oka studied pressure loss of blood flow through tapered tubes.

Hence in order to have a better understanding of the complex nature of blood flow, one would not only take in to account the properties of blood but also the geometrical shape of its vessels. In present chapter, we have studied the flow of blood through uniform tapered tube assuming that it obeys the casson fluid model. The expressions for pressure gradient and wall shear stress have been obtained. Variations of pressure gradient and shear stress at the wall have been studied for constant suspension concentration with respect to tapered angle over the flow rate range 0.01 to 0.1 cc/ sec Numerical values of pressure gradient for Bingham, Power law and Newtonian fluids have been obtained for fixed value of suspension concentration.

3.2 Mathematical Model



Geometry of the vessel
Fig 3.1

Let us consider a steady laminar flow of an incompressible Non-Newtonian, casson fluid in a uniformly tapered vessel is shown in above fig 3.1.The geometry is most conviniently represented by the cylindrical Coordinates: The radius R (z) of the tapered tube is given by

$$R(z) = R_0 - z \tan \theta \qquad (1)$$

where R_0 is the radius at Z=0, θ is the tapered angle and Z is along the axis of the tapered tube.The problem has been studied under the following assumptions.

(1) The motion has an axial symmety,

(2) . No body forces act in the fluid.

(3) The motion is so slow that the inertia term can be neglected.

(4) The tapered angle is very small.

The equations of motion and continuity for a well developed steady viscous incompressible laminar flow under no body forces in the cylindrical coordinates (r, θ, z) Whose origin lies on the axis of the vessel provide

$$O = -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) \qquad (2)$$

$$O = \frac{\partial p}{\partial r} \qquad (3)$$

$$and \frac{\partial v}{\partial z} = 0. \qquad (4)$$

where P, τ_{rz} and V are respectively pressure ,shear stress normal to r in z direction and the axial velocity.

The casson constitutive equation is given as

$$\tau^{\frac{1}{2}} = \tau y^{\frac{1}{2}} = \eta^{\frac{1}{2}} y^{\frac{1}{2}} \qquad \tau \geq \tau y \qquad (5)$$

$$\dot{\gamma} = o, \qquad \tau \leq \tau y,$$

where $\dot{y} = \frac{dv}{dr}$, τy and μ represent shear strain, yield stress, coefficient of viscocity repectively.

The oppropriate boundary conditionsare given by

V =o at r = R (z) (6)

τ_{rz} = tw at r = R (z) (7)

V = V_y at r = R_y (8)

and τ_{rz} is finite are r =o (9)

Where R_y is the plug radius and Vy is the plug velocity.

3.4 The Mathematical Analysis

In the analysis to follow, it is assumed that the pressure gradient is a function of axial coordinate only. Then with the

help of integration of equ. (2) with boudary conditions, yields further,

$$\tau_{rz} = \frac{r}{z}\frac{\partial p}{\partial z} \tag{10}$$

From equ. (5) & (10) we have

$$\tau_0^{\frac{1}{2}} + \eta^{\frac{1}{2}}\gamma^{\frac{1}{2}} = \left(\frac{\Delta p}{2L} r\right)^{\frac{1}{2}}$$

Or $$\gamma^{\frac{1}{2}} = \left(\frac{1}{\eta}\right)^{\frac{1}{2}}\left(\frac{\Delta p}{2L} r\right)^{\frac{1}{2}} - \tau_0^{\frac{1}{2}}$$

hence

$$\frac{dv}{dr} = \frac{1}{\eta}\left[\left(\frac{\tau\varpi}{R(z)} r\right)^{\frac{1}{2}} - \tau_0^{\frac{1}{2}}\right]^2 \tag{11}$$

Here L is the length of the tapered tube. Integration of (II) with the help of boundary conditions, we get

$$v = \frac{\tau\varpi}{2\eta} R(z)\left[1 - \frac{r^2}{R^2_{(2)}} - \frac{8}{3}\beta^{\frac{1}{2}}\left(1 - \frac{r^{\frac{3}{2}}}{R^{\frac{3}{2}}_{(2)}}\right) + 2\beta\left(1 - \frac{r}{R_0}\right)\right] \tag{12}$$

Where $\beta = \left(\frac{\tau_y}{\tau_w}\right)$

On substituting $r = \beta R(z)$, the plug velocity is given by

$$V_y = \frac{\tau_w}{2} R(z)\left[1 - \frac{8}{3}\beta^{\frac{1}{2}} + 2\beta - \frac{1}{3}\beta^2\right] \tag{13}$$

Volumetric Flow Rate

The volume flow rate Q is given by

$$Q = Q_y + Q^1 \tag{14}$$

Where $Q_y = \int_{0}^{R_y} 2\pi V_y r dr$

$$= 2\pi V_y \frac{R_y^{\,2}}{2} = \pi V_y R_y^{\,2} \qquad (15)$$

and $Q^1 = \int_{R_y}^{R_{(z)}} 2\pi V r dr \qquad (16)$

Now substituting for V and Vy from equ.(12) & (13) into equations (15) & (16) respectively, we get on neglecting the higher powers

$$Q_y = \frac{\pi \tau w}{2\eta} R^3{}_{(z)} \beta^2 \left[1 - \frac{8}{3}\beta^{\frac{1}{2}} + 2\beta - \frac{1}{3}\beta^2\right] \qquad (17)$$

and

$$Q^1 = \int_{R_y}^{R_{(z)}} 2\pi \frac{\tau\varpi}{2\eta} R(z) \left[1 - \frac{r^2}{R^2{}_{(z)}} - \frac{8}{3}\beta^{\frac{1}{2}}\left(1 - \frac{r^{\frac{3}{2}}}{R^3{}_{(z)}}\right) + 2\beta\left(1 - \frac{r}{R(z)}\right)\right] rdr$$

Hence

$$Q^1 = \frac{\pi\tau\varpi}{\eta} R^3{}_{(z)} \left\{\frac{1}{2} - \frac{4}{7}\beta^{\frac{1}{2}} + \beta - \frac{\beta^2}{2}\right\}$$

The pressure gradient is obtained from equ. (10) & (19) with the help of boundary condition (6) as

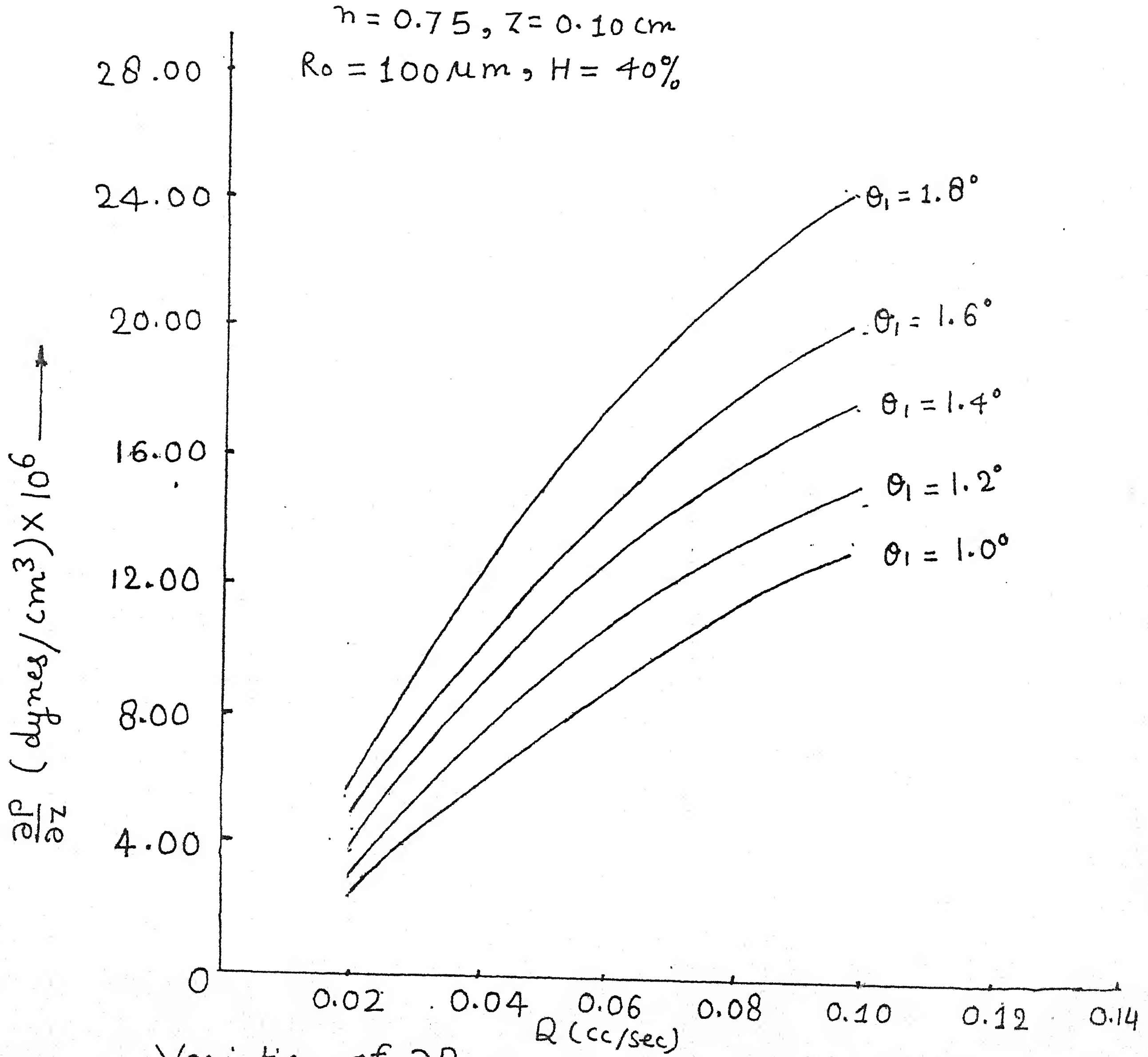
$$\frac{\partial p}{\partial z} = \frac{4\eta Q}{\pi R^4{}_{(z)}} \left[1 - \frac{8}{7}\beta^{\frac{1}{2}} \underset{-\beta}{} - \frac{5}{6}\beta^2\right] \qquad (20)$$

The shear stress at the wall is given by equ. (19) as

$$\tau\varpi_{(z)} = \frac{2\eta Q}{\pi R^3(z)} \left[1 + \frac{8}{7}\beta^{\frac{1}{2}} - \beta - \frac{5}{6}\beta^2\right] \qquad (21)$$

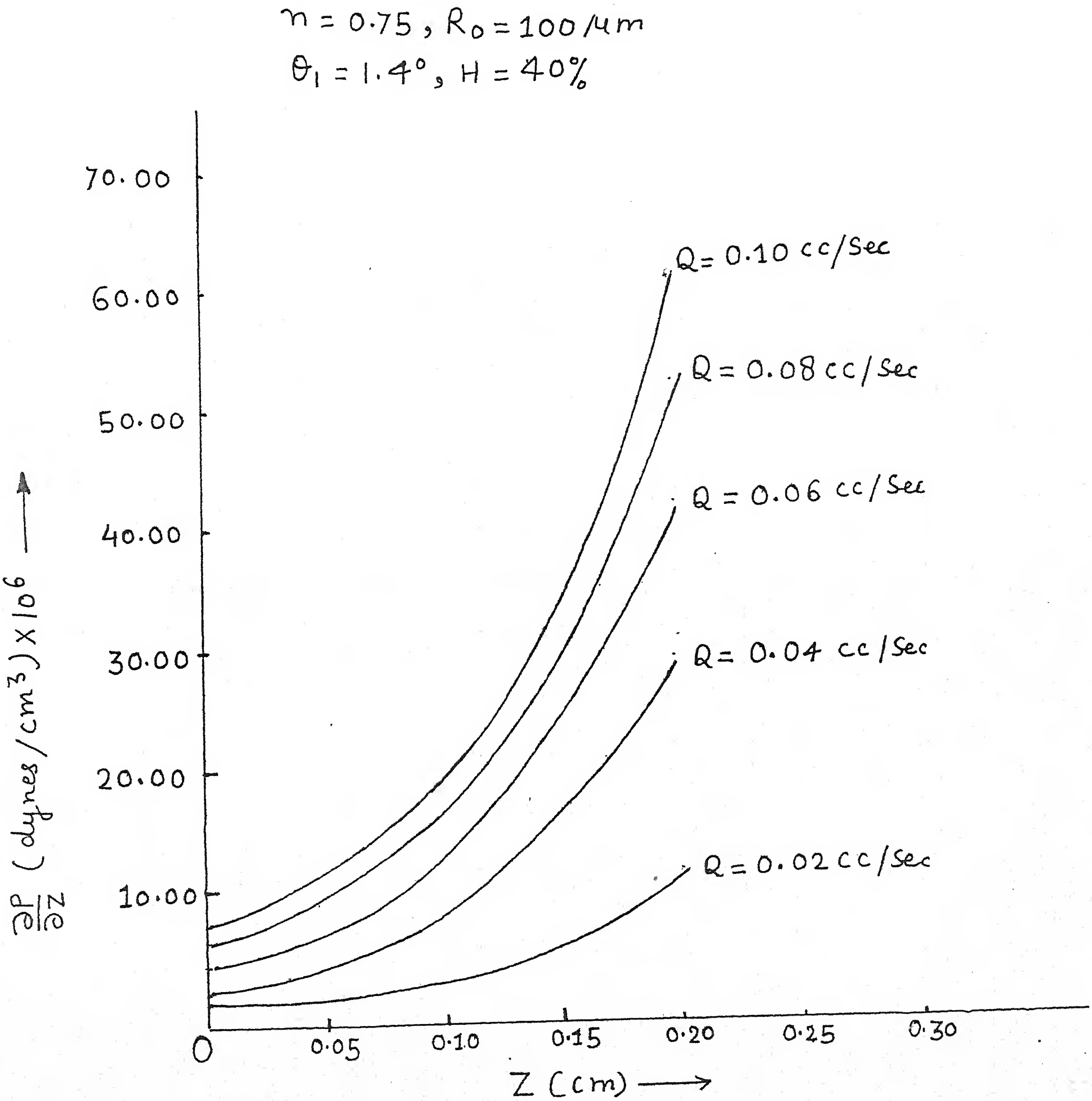
Finally equ's (20) & (21) yield

$$\tau\varpi_{(z)} = \frac{R(z)}{2}\frac{\partial p}{\partial z} \qquad (22)$$



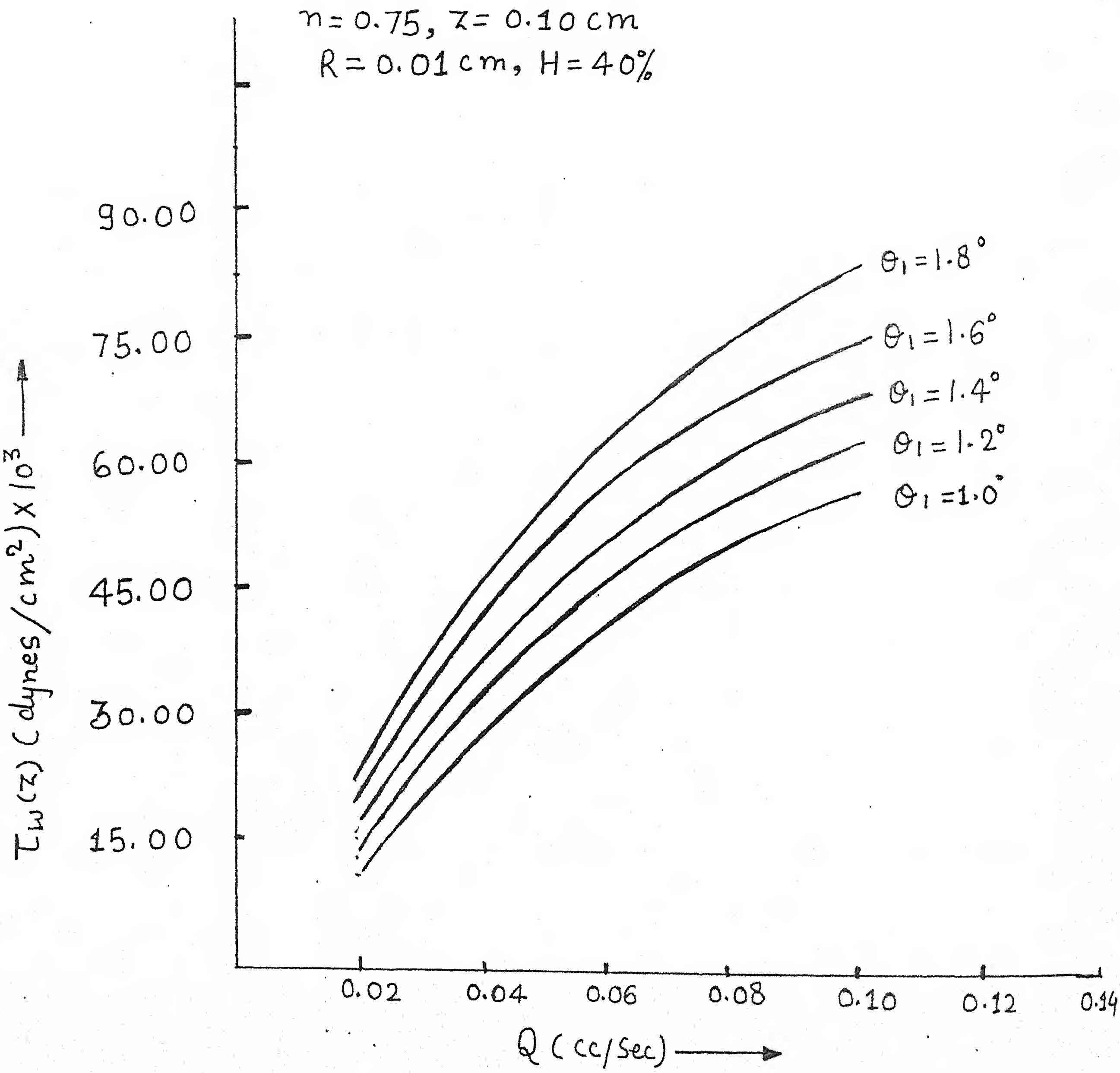
Variation of $\frac{\partial P}{\partial z}$ with Q for different taper angle

Figure 3.1



Variation of Pressure gradient with axial distance (z)

Fig 3.2



Variation of $\tau_w(z)$ with Q for different taper angle

Fig 3.3

Results and discussions -

It may be noted from equations (20) and (21) that the pressure gradient and the shear stress at the wall are not constant along the tube axis.

Infact, they increase with the decrease in the radius of the tapered tube. From equations (22) it is observed that the shear stress at the wall depends on the pressure gradient. Dwivedi et al have proposed the micropolar fluid madel for blood flow through a small taperd tube, where they have assumed the existence of a constant pressure gradient throughout the analysis. It appears to be incorrect for the reasons prescribed above.

The variations of pressure gradient and shear stress at the wall are computed from egus (19) to (21) with flow rate over the range .01 to .1 cc/sec. for different taperd angle amd defferent axial destance. It is observed from fig3.1&3.2 that the pressure gradient increases as the tapered angle and axial distance increases, flow rate being taken over the range 0.01 to 0.1 cc/sec Figure .....3.3.. shows the variation of shear stress at the wall with flow rate for defferent tapered angles. We find that $\tau w_{(z)}$ increases with the increase in suspension concentration and tapered angle.

$\tau w_{(z)}$ is also an increasing function of axial distance. Therefore for known flow rate, the shear stress can be computed at any point of the tapered tube.

Tables..3.1.... (a, b) show that for Bingham model $\frac{\partial p}{\partial z}$ increases with the increase in tapered angle and axial distance for a fixed suspension concentration 40% however, these values are comparatively less than those for casson model.

Few Newtonian fluid, through $\frac{\partial p}{\partial z}$ has the same trend as for casson. its values smaller than those of casson model. It is hoped that the present model will provide a better understanding of vascular fluid mechanics.

Table 3.1

Bingham Fluid (n=1)

(a) Variation of $\frac{\partial p}{\partial z}$ with Q for defferent θ

H=40% R_o = .01 cm Z = .10 cm

$\frac{\partial p}{\partial z}$ (dynes/cm^3) x 10 ^{6}

Q(cc/sec)	θ=1.0°	1.2°	1.4°	1.6°	1.8°
0.02	0.537	0.633	0.755	0.901	1.097
0.04	1.096	1.295	1.540	1.840	2.283
0.06	1.634	1.930	2.295	2.793	3.336
0.08	2.193	2.590	3.080	3.682	4.477
0.10	2.736	3.233	3.844	4.595	5.588

References -

1] Iida and Murata, 1980: Biorheology 17, P.377

2] Arimanetal, 1974: J Appl. Mesch. 41 P1.

3] Block, E.H, 1962: Amer .J. Anat. 110, P.125

4] Dwivedi et al, 1982: Ind. J. Technol. 20, P. 295

5] Oka, s, 1964 : J phys. Soc. Japan 19, P 1481

(b) Variation of $\frac{\partial p}{\partial z}$ with Z for different Q

H= 40%, Ro = 0.01Cm θ = 1.4

$\frac{\partial p}{\partial z}$ (dynes/cm^3) x 10 ^{6}

Q	Z=0	0.05	0.10	0.15	0.20
.02	0.243	0.414	0.755	1.571	3.559
.04	0.497	0.845	1.541	3.085	7.262
.06	0.741	1.260	2.297	4.998	10.792
.08	0.994	1.691	3.083	6.171	14.524
.10	1.241	2.110	3.848	7.701	18.127

Table 3.2
Power Law Fluid

(a) Variation of $\frac{\partial p}{\partial z}$ with R for different $\theta°$

H=40%, R_o = 0.01cm Z=.01cm n= .75

Q	θ=1.0°	1.2°	1.4°	1.6°	1.8°
0.02	2.062	2.377	2.726	3.216	3.775
0.04	3.107	5.893	6.760	7.968	9.348
0.06	6.899	7.961	9.132	10.764	12.128
0.08	8.556	9.874	11.326	13.350	15.662
0.10	10.124	11.684	13.401	15.797	18.532

(b) Variation of $\frac{\partial p}{\partial z}$ with Z for different Q

Q	z=o	0.05	0.10	0.15	0.20
0.02	1.104	1.706	2.740	4.429	97.736
0.04	3.736	4.218	6.794	10.944	24.168
0.06	3.696	5.698	9.178	14.784	32.648
0.08	4.584	7.068	11.383	19.336	40.492
0.10	5.424	8.362	13.469	21.696	47.912

Table 3.3
Newtonian Fluid

(a) Variation of $\frac{\partial p}{\partial z}$ with Q for different θ

H=40%, Ro=.01 cm Z=.10cm

Q	θ =1°	1.2°	1.4°	1.6°	1.8°
0.02	0.418	0.496	0.589	0.703	0.853
0.04	0.857	1.009	1.203	1.428	1.744
0.06	1.269	1.504	1.793	2.128	2.594
0.08	1.703	2.019	2.407	2.850	3.488
0.10	2.125	2.520	3.004	3.564	4.313

(b) Variation of $\frac{\partial p}{\partial z}$ for different values of Q and Z

Q	Z=0	0.05	0.10	0.15	0.20
0.02	0.232	0.312	0.411	0.512	0.523
0.04	0.435	0.488	0.512	0.612	0.711
0.06	0.812	0.912	1.008	1.111	1.212
0.08	1.805	1.015	1.189	1.212	1.314
0.10	2.004	2.810	2.981	3.005	3.502

Table 3.4
Casson Fluid

(a) Variation of $\frac{\partial p}{\partial z}$ with Q for different $\theta°$

H=40%, Ro =0.01 cm Z =1.4

Q	$\theta°=1°$	1.2°	1.4°	1.6°	1.8°
0.02	3.001	3.278	3.415	3.912	4.775
0.04	4.201	6.022	6.819	7.112	8.992
0.06	7.208	9.002	11.178	13.527	15.819
0.08	9.552	11.553	13.112	15.778	17.111
0.10	10.812	12.212	14.013	16.123	18.412

(b) Variation of $\frac{\partial p}{\partial z}$ with Z for different Q

β =0.01, R_o=.01, θ =1.2°, μ = 0.1

Q	Z=o	0.05	0.10	0.15	0.20
0.02	0.280	0.303	0.338	0.382	0.44
0.04	0.560	0.607	0.676	0.765	0.88
0.06	0.840	0.910	1.015	1.148	1.32
0.08	1.121	1.213	1.353	1.530	1.76
0.10	1.401	1.572	1.692	1.913	2.2

Table 3

(c) Variation of $\frac{\partial p}{\partial z}$ with $\tau_w(z)$

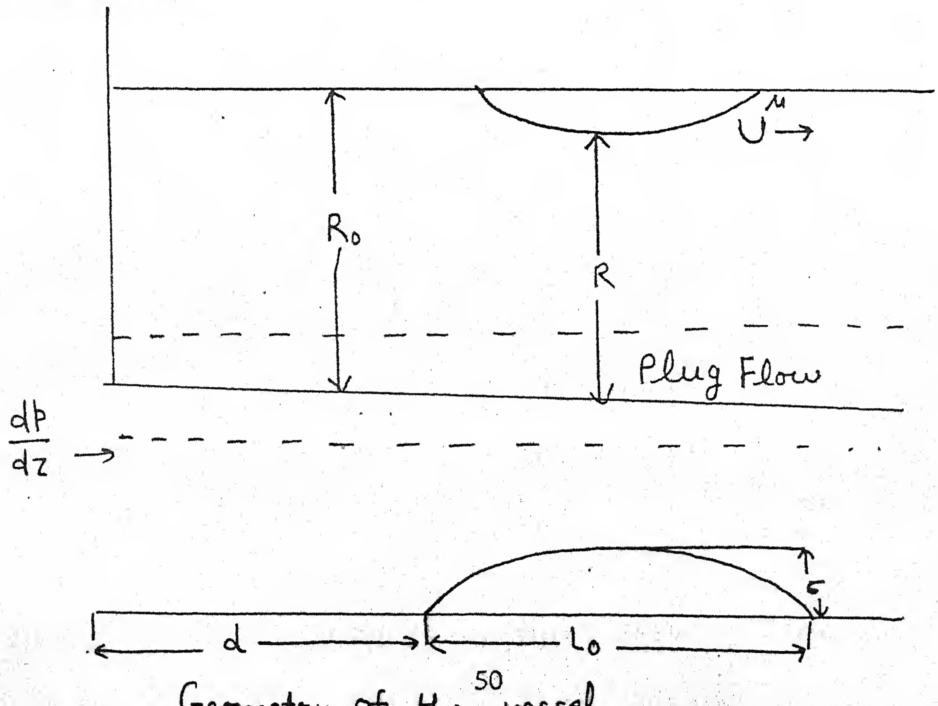
R_o=.01, Z=.0

$\tau_w(z)$	$\theta°=1°$	1.2°	1.4°	1.6°
0.02	4.848	5.063	5.263	5.556
0.04	9.696	10.127	10.526	11.111
0.06	14.545	15.189	15.789	16.667
0.08	19.394	20.253	21.052	22.22
0.10	24.292	25.316	26.316	27.778

Chapter IV

Effect of Mild - Stenosis on Blood Flow Through Narrow Vessels

4·1 Introduction-



Geometry of the vessel

Fig 4.1

A stenosis causes the narrowing of the bloodvessels due to the developement of abnormal tissue and it gives way to serious circulatory disorder by reducing or occluding the blood supply. It is well known that abnormal growth in the lumen of the arterial wall develope at various locations of the cardio-vascular system under diseased condition.

Arteriosclerosis or stenosis, as it is called, is one of the most widespread arterial diseases. The actual cause of stenosis is not well known but its effects on the cardiovascular system can be understood by studying the flow behaviour and flow characteristics in its vicinity. Several theoretical and mathematical attempts have been made to study the blood flow characterists due to the presence of a stenosis in the lumen of a blood vessel.

Assuming blood to be a newtonian fluid many authors like chakravarty and chawdary [22], Doffin and Chagneaw [41] Liou ltal [79], Mac Donald [82], Mann etal. [83], Mehrotra and Jayaramah [88], Misra and chakravaty [92], Morgan and young [93], pollard [112], young [176] and young etal. [178] studied the blood behaviour in stenosised tube.

Mehrotra and Jayaraman [88] have studied steady laminar flow of blood in an elliptic stenotic tube by assuming blood as Newtonian fluid and analytical expressions for shear stress distrubution and impedance are obtained. Misra and Chakravarty [92] studied blood flow through an arterial segment having a stenosis and the artery is modelled as an initially stressed erthotropic elastic tube. They have assumed that blood is a

newtonian fluid In small vessels and at low shear rates the flow behaviour isNot Newtonian, and it has been seen that the velocity profiles are not parabolic under these flow conditions. . Bugliarello et al (16), Randetal (117) and many others have consideraed the non Newtonian behaviourS of blood in their studies. Recently sinhaand Singh [130], Srivastava [138] studied the effect of stenosis on blood flow, through the couple stress fluid model of blood.

Bitoun and Bellet [7], Rabinovets et. al, Slazback etal. [134], and Tandon et al [150] have studed pulsatile flow of blood through rigid and pulsating stenoses. [illegible] The geometry of constriction chosen was defined by a Gaussion normal destribution curve. young and Tsai [177] studied some of the flow characteristics in models of arterial stenosis under steady and pulsatile flow conditions. These experiments have yielded, in the case of steady flow, a description of the extent of separated flow regions, a measure of pressure losses of constriction. Several other workers studied the characteristics of blood flow due to presence of stenosis in the lumen of the artery [Abdellah and Hwang] [1], Chakravarty [20], Devanathan and Parvathamma [37], Nakameera and Sawada [95], Pralhad and schultz [114] sud and sekhon [141] and yamaguchi and hanai [147] by assuming different models of blood, Further, Shukla et al [127] studied the effect of Peripheral layer viscocity of blood on the flow characteristics with mild stenoses.Shukla et al[128] investigated the effects of stenosis on peripheral resistance and wall shear stress in an artery by considering the blood as power law and casson's fluid. But it has been reported by scott

- Blair and Spanner [148] that the hershel - bulkey fluid model of blood has an edye over other fluid models. Therefore studying the effect of an axially symmetric mild - stenosis on the flow of blood in narrow vessels, we assume that blood obeys hershel - Bulkley constitutive equation of motion. The volume flow rate, apparent fluidity of blood and shearing stress at the wall in the stenosized tube are discussed with respect to different field parameters.

P.T.O

4.2 The Mathematical Models -

In the present chapter we analyse a Herschel-Bulkey fluid model of blood flowing through narrow vessels with mild stenosis, cylindrical coordinates system (r, z, θ) has been considered with z - axis coinciding with the central line of the vessel. The geometry of the stenosis as in fig...4.1.... is described as

$$\frac{R}{R_o} = 1 - \frac{\sigma}{2R_o}\left[1 + \cos\frac{2\pi}{L_o}\left(z - d - \frac{L_o}{2}\right)\right]$$

$$d \le z \le L_o + d \qquad (1)$$

$= 1$, otherwise

We shall assume further that

$$\text{(a)}\ \frac{\sigma}{R_0} << 1,\ \text{(b)}\ \frac{Ro}{Lo} \sim 0(1),\ \text{(c)}\ \mathrm{Re}\left(\frac{\sigma}{L_0}\right) << 1 \quad (2)$$

Where R_O is the radius of unobstructed tube, R the radius of obstructed tube, L_O the length of the stenosis, d the location of stenosis, the maximum height of stenostic growth and Re is the Reynolds number of the flow.

4.3 The Governing Equation-

The equation of motion and continuity for steady incompressible laminar flow under no body forces are given by

$$\frac{\partial P}{\partial Z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) \quad (3)$$

$$0 = \frac{\partial p}{\partial r} \quad (4)$$

$$\frac{\partial v}{\partial z} = 0 \quad (5)$$

Where P is the pressure, τ_{rz} the shear stress normal to r in z direction, v the axis velocity.

The constitutive equation in one-diamensional form of Hershel-Bulkey fluid is

$$\tau = \mu\gamma^n + \tau_0 : \tau \geq \tau_0$$

$$\dot{y} = 0 : \tau \leq \tau_0 \tag{6}$$

Where τ represents shear stress, τ_0 the yield stress, μ the coefficient of viscocity and $\dot{\gamma} = \left(\frac{dv}{dr}\right)$ the strain rate.

4.4 The Boundary Conditions -

The appropriate boundary condition are given by

$$v=0, \quad \tau_{rz} = \tau_w \quad \text{at } r=R \tag{7}$$

$$\tau \text{ is finite at } r = 0 \tag{8}$$

Where τ_w is the shear stress at the stenotic wall

4.5 The Mathematical Analysis -

Integrating equation (3) we get

$$r\tau_{rz} = \frac{\partial p}{\partial z}\frac{r^2}{2} + c \tag{9}$$

Where C is constant of integration

Using the boundry condition (8) in equ. (9) we get -

$$\tau_{rz} = \frac{\Delta P}{2L} r \tag{10}$$

Where ΔP is the pressure drop over a lenght L of the tube.

From equation (6) with boundary condition (7) we get -

$$V = \frac{n}{(n+1)} \left(\frac{\tau_w}{\mu}\right)^{\frac{1}{n}} R \left[(1-\rho)^{\frac{n+1}{n}} - \left(\frac{r}{R} - p\right)^{\frac{n+1}{n}} \right] \tag{11}$$

Where $P = \frac{\tau_0}{\tau_w}$

Plug flow exist whenever the shear stress does not exceed yield stress and the plug velocity Vy can be obtained by putting r=PR in equ (11)

$$V_y = \frac{n}{(n+1)} \left(\frac{\tau_w}{\mu}\right)^{\frac{1}{n}} R (1-P)^{\frac{n+1}{n}} \tag{12}$$

From equ. (11) we can obtain velocity equation for Bingham and power law fluids by putting r=1 and $\rho = 0$ respectively.

4.6 The Volume Flow Rate -

The volume flow rate Q is given by

$Q = Q_y + Q'$

Where

$$Q_y = \int_0^{R_y} 2\pi V_y r dr = \pi V_y R_y^{\,2} \qquad (14)$$

$$Q' = \int_{R_y}^{R} 2\pi v r dr \qquad (15)$$

Using equation (11) & (12) in (14 & (15)

$$Q_y = \frac{n\pi}{(n+1)}\left(\frac{\tau_w}{\mu}\right)^{\frac{1}{n}} R_o^{\,3} p^2 (1-p)^{\frac{n+1}{n}} \left(\frac{R}{R_o}\right)^3 \qquad (16)$$

$$Q^1 = \frac{n\pi}{n+1}\left(\frac{\tau W}{\mu}\right)^{\frac{1}{n}} R_0^{\,3}\left[(1-P^2)(1-P)^{\frac{n+1}{n}} - \frac{2n}{(2n+1)}(1-p)^{\frac{2n+1}{n}}\right]$$

$$+\frac{2n^2}{(2n+1)(3n+1)}(1-p)^{\frac{3n+1}{n}}\left(\frac{R}{R_o}\right)^3 \qquad (17)$$

Hence for equ. (12), (16) & (17) we have

$$Q = \frac{n\pi}{3n+1}\tau_w^{\frac{1}{n}} R_0^{\,3} \frac{1}{\mu^{\frac{1}{n}}}\left[1-\frac{3n+1}{n(2n+1)}\rho\right]\left(\frac{R}{R0}\right)^3 \qquad (18)$$

The non-diamensional flow rate $\overline{Q}$ is given by

$$\frac{Q}{Q_0} = \overline{Q} = Q_0\left[1-\frac{3n+1}{n(2n+1)}\rho\right]\left(\frac{R}{R_0}\right)^3 \qquad (19)$$

Where

$$Q_0 = \frac{n\pi}{(3n+1)}\left(\frac{\tau_w}{\mu}\right)^{\frac{1}{n}} R_0^{\,3}$$

At the maximum height of stenoses the volume flow rate $\overline{Q}$ is given by

$$\bar{Q} = \left[1 - \frac{3n+1}{n(2n+1)}\beta\right]\left(1 - \frac{3\sigma}{R_0}\right) \tag{20}$$

higher orders of $\frac{\sigma}{R_0}$ are neglected.

4.7 Apparent Fluidity and Shear Stress at the Wall -

From equ. (18) the apparent floidity ϕa is given by -

$$\phi a = \left[\frac{(3n+1)Q}{n\pi\tau_w^{\frac{1}{n}} R_0^{\ 3}}\right]^n$$

$$\text{or} \quad \phi a = \frac{1}{\mu}\left[1 - \frac{(3n+1)}{(2n+1)}\beta\right]\left(\frac{R}{R_0}\right)^{3n} \tag{21}$$

higher powers of β are neglected. At the meximum height of stenosis i.e. at $z = d + \frac{L_0}{2}$ we have

$$\phi a = \frac{1}{\mu}\left[1 - \frac{3n+1}{2n+1}\beta\right]\left(1 - 3n\frac{\sigma}{R_0}\right) \tag{22}$$

From equation (18), the shear stress τ_w at the wall is given by

$$\tau_w = \left[\frac{(3n+1)Q\mu^{\frac{1}{n}}}{n\pi R_0^{\ 3}}\right]^n \left(1 + \frac{3n+1}{2n+1}\beta\right)\left(\frac{R}{R_0}\right)^{-3n} \tag{23}$$

Now the wall shear stress τ_w at the maximum

height of stenosis i.e. at $z = d + \frac{L_0}{2}$ is obtained, as

$$\tau_w = \left[\frac{(3n+1)Q\mu^{\frac{1}{n}}}{n\pi R_0^{\ 3}}\right]^n \left(1 + \frac{3n+1}{2n+1}\beta\right)\left(1 + 3n\frac{\sigma}{R_0}\right) \qquad (24)$$

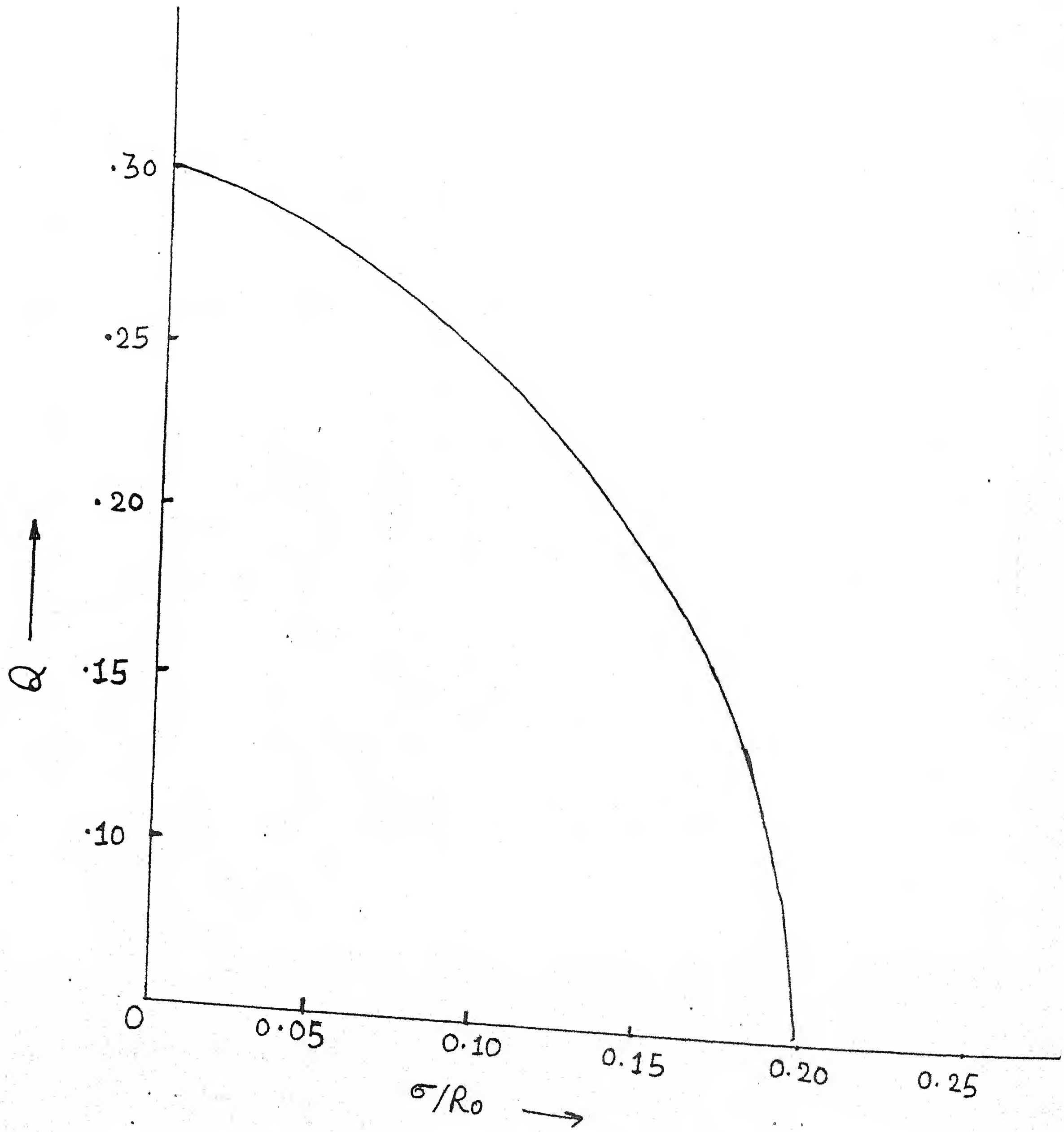
In non- dimensional form, we have

$$\bar{\tau}_w = \left(1 + \frac{3n+1}{2n+1}\beta\right)\left(1 + 3n\frac{\sigma}{R_0}\right) \qquad (25)$$

$$\bar{\tau}_w = \frac{\tau_w}{\tau'}$$

$\tau' = \left[\frac{(3n+1)Q\mu^{\frac{1}{n}}}{n\pi R_0^{\ 3}}\right]^n$, τ' being the Wall shear stress for

-no stenosis case.



Variation of Q with stenosis height

Fig 4.2

Results and Discussion -

In the present chapter a non - newtonian Herschel - Bulkley fluid model has been studied in the presence of mild stenosis Analytical expressions for volume flow rate, apparent fluidity and wall shear stress at the maximum height of stenosis have been obtained Further, it is observed that in the earliar models including the two fluid models of Shukla et al [126] the blood has earliar been assumed to be a Newtonian fluid or has been modelled as double layered Newtonian fluid with different viscosities in the core and the peripheral region But erperimental results (Cokelet [34] and Goldsmith and skalak [53] clearly indicate that blood exhibits non - Newtonian effects and also indicate (for flow through small diameter tubes) the existence of a peripheral layer of Newtonian fluid and core of non - Newtonian fluid (RBC suspension) In our analysis it is assumed that blood is non Newtonian and flow is single fluid model. The results are discussed with stenosis height, yield stress of blood and non - Newtonian parmeter n

Figures ..4.2.. and ....4.3..... depict the variation of volume flow rate $(\bar{Q})$ with stenosis height $\left(\frac{\sigma}{R_0}\right)$ and yield stress (β) for different values of n (=0.5,

1.0, 1.5). The plot for n=1.0 is for Bingham fluid. From figures it is clear that Q decreases as $\frac{\sigma}{R_0}$ and β increases for fixed value of n. The values of Q are in increasing order with parameter n. From results it is observed that the flow rate is decreased from Newtonian case by 25%, for n =.5, 13%, for n=1; 7% for n=1.5 for fixed $\frac{\sigma}{R_0}$ and β. For $\frac{\sigma}{R_0}$ =0.10, the values of Q are decreased by 30% from those for no stenose case for fixed n and β.

Table 4.1 and 4.2 shows the variation of apparent fluidity ϕa with σ/R_0 for different value of β & n(=.5,1.0 1.5). From graphs we observe that apparent fluidity decreases as stenoses height and yield stress increases for fixed n. The value decreases uniformly with parameter n. From numerical resultes, we abtain that for $\frac{\sigma}{R_0}$ = .10, the values of ϕa are decreased by 15% for n=0.5, 30% for n=1.0; 45% for n=1.5 from those for no stenosis case. Further we observe that ϕa is decreased by 13% for n=1.0, β=0.10 as compared to Newtonian value. On comparing the results with Newtonian we find that at a fixed stenosis height, ϕ_a is greater than that for Newtonian value for n<1 and smaller for n≥1.

Tables (4.2)....... and ..(4.4)..... depict the variation of wall shear stress $\bar{\tau}_w$ with $\frac{\sigma}{R_w}$ and β for different values of β & n (=0.5, 1.0, 1.5). From figures, it is seen that $\bar{\tau}_w$ increases with respect to $\frac{\sigma}{R_0}$ and β for fixed n. The valus of τ_w are in increasing order with n. The results show that the value of τ_w is increased by 17% (approx) for n=0.5, $\frac{\sigma}{R_0}$=0.10; 30% for n=1.0, $\frac{\sigma}{R_0}$=0.10; 45% for n=1.5, $\frac{\sigma}{R_0}$ =0.10 for no - stenosis case Also we obtain that $\bar{\tau}_w$ is increased by 13% from Newtonian case for n = 1.0, β=0.10. Srivastava [136] reported that for couple stress, fluid wall shear stress increased by 60% from no - stenosis case it is noted that the values of wall shear stress are greater than Newtonian value for $n \geq 1$ and smaller for $n<1$. Our results of $\bar{\tau}_w$ for $n \geq 1$ are in conformity with the results of Srivastava [136] for couple stress fluid model.

From equ. ....22.............and........24.......we have calculated the numerical values of ϕa and $\bar{\tau}_w$ for power law fluid (β=0) (Table ..1. & .2...). From these tables, we abserve that ϕa decreases with increase in $\frac{\sigma}{R_0}$ and n, and $\bar{\tau}_w$ increases with respect to $\frac{\sigma}{R_0}$ and n.

Table -4.1

Variation of ϕa with $\frac{\sigma}{R_0}$ for different n

$\frac{\sigma}{R_0}$	ϕa			
	n	0.5	1.0	1.5
0.5		0.421	0.387	0.353
0.10		0.387	0.319	0.250
0.15		0.353	0.250	0.148
0.20		0.319	0.182	0.046

Table 4.2

Variation of $\overline{\tau_w}$ with $\frac{\sigma}{R_0}$ for different n

$\frac{\sigma}{R_0}$	$\overline{\tau_w}$			
	n	0.5	1.0	1.5
0.05		1.075	1.150	1.225
0.10		1.150	1.300	1.450
0.15		1.225	1.450	1.675
0.20		1.300	1.600	1.900

TABLE 4.3

VARIATION OF ϕ_a WITH σ/R_0 FOR DIFFERENT β

σ/R_0	ϕ_a			
	$\beta =$ ·10	·15	·20	·25
·05	0·257	0·177	0·102	0·015
·10	0·236	0·163	0·089	0·014
·15	0·215	0·148	0·081	0·013
·20	0·194	0·134	0·073	0·012

TABLE 4.4

VARIATION OF τ_w WITH σ/R_0 FOR DIFFERENT β

σ/R_0	τ_w			
	$\beta =$ ·10	·15	·20	·25
·05	1·501	1·697	1·904	2·111
·10	1·539	1·816	2·037	2·259
·15	1·698	1·934	2·169	2·406
·20	1·802	2·053	2·302	2·553

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CHAPTER V

A Study of Bingham Fluid Model of Blood Flow in Capillaries

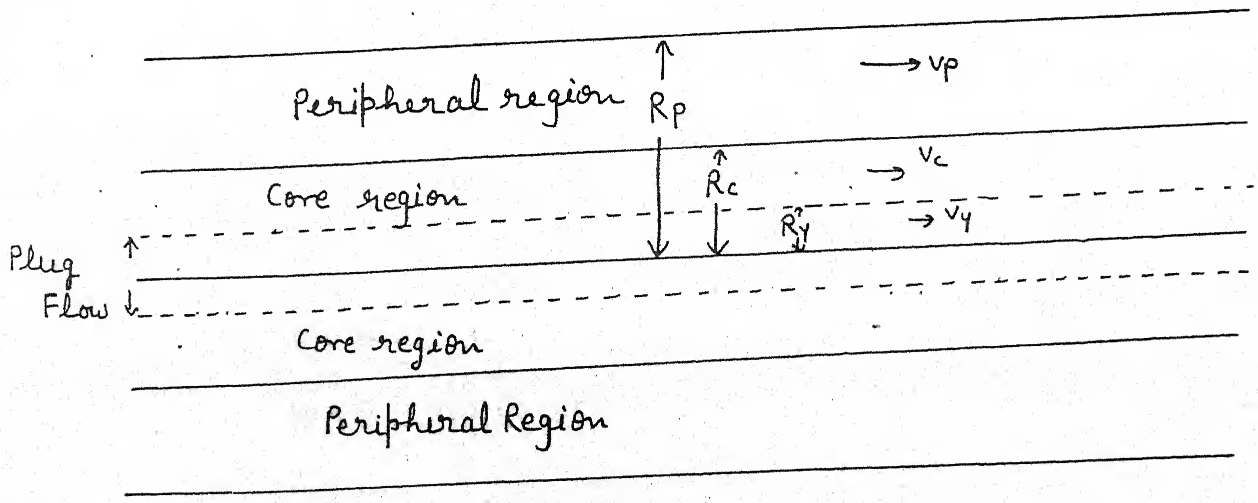
Blood flow in capillaries of internal deameter equal to that of red cell is quite different from that in the large vessels. Since the size of RBC is not negligible as compared to vessels diameter, it is necessary to consider the flow of blood in capillaries as two phase non homogeneous flow.

The apparent viscocity of blood depends on several factors such as plasma viscosity, hematocrit, size of the vessel, shear rate, rate of flow, regidity and deformability etc. Also the apparent viscosity in

capillaries is much lower than in large vessels (Dentifass). Also it is well known that apparent viscosity of blood decreases as the tube radius decreases. (Fahraeus lindguist effect). Gupta and Seshadri, Lida and Murata, Charm et al have considered the two fluid models, in which both layers (Peripheral Plasma layer and core hematocrit layer) are of Newtonian fluids with different viscosities and yield stresses.

In the present paperwe have considered two layer fluid model of blood with no slip velociity at the wall, both satisfying Bingham constitutive equation. In this paper velocity field, volume flow rate and apparent fluidity have been found. Flow parameters have been explained.

5.1 The Mathematical Analysis -



The flow of blood is axially symmetric and is in the Z direction.

The equation of motion and continuity for steady, incompressible, laminar viscous flow under zero body forces are -

$$\frac{\partial v}{\partial z} = 0 \tag{1}$$

$$\frac{-\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) = 0 \tag{2}$$

$$\frac{\partial p}{\partial r} = 0 \tag{3}$$

Where τ is shear stress normal to r in z direction, P is the pressure and V is velocity.

Let a two - fluid model of blood with a central core region of radius Rc and a plasma layer of thickness d = (R_p -R_c), satisfying Bingham plastic constitutive equation.

$$\tau = \tau_y + \mu\dot{\gamma}, \quad \tau > \tau_y \tag{4}$$

$$\dot{\gamma} = 0, \quad \tau \le \tau_y$$

Where μ is Newtonian viscosity, τ is shear stress, τ_y is yield stress, $\dot{\gamma}$ is shear strain.

Boundary conditions are

$\tau p = \tau c$, Vp=Vc at r=Rc

$Vp=0$ at $r=Rp$ (5)

$\dot{\gamma}=0$, $\tau_c=\tau_y$, $Vc=Vy$ at $r=Ry$

τ is finite at $r = 0$

Where Ry = Plug flow radius, Vy = Plug flow velocity. suffixes P and C denote the value of the corresponding parameter in peripheral layer and core layer, respectively.

From (2) & (5) we obtain

$$\tau=\frac{r}{2}\frac{\partial p}{\partial z} \tag{6}$$

Now using (6) in equ. (4)

we have -

$$\frac{1}{\mu}\left(\frac{r}{2}\frac{\partial p}{\partial z}-\tau_y\right)=\frac{\partial v}{\partial r} \tag{7}$$

$$\frac{\partial vp}{\partial r}=-\left(\frac{1}{\mu p}\right)\left(\frac{dp}{\partial z}\frac{r}{2}-\tau p\right), R_c \le r \le R_p \tag{8}$$

$$\frac{\partial vc}{\partial r}=-\frac{1}{\mu_c}\left(\frac{\partial p}{\partial z}\frac{r}{2}-\tau_c\right), R_y \le r \le R_c \tag{9}$$

$$\frac{\partial V_y}{\partial r}=0, 0 \le r \le R_y \tag{10}$$

Integrating equations (8), (9) and (10)

we get -

$$V_p=\int-\frac{1}{\mu_p}\left(\frac{\partial p}{\partial z}\frac{r}{2}-\tau_p\right)dr$$

or

$$V_p = \frac{1}{\mu_p}\frac{\tau_w R_p}{2}\left[1 - \frac{r^2}{R_p{}^2} + 2\alpha_p\left(\frac{r}{R_p} - 1\right)\right] \tag{11}$$

Similarly

$$V_c = \frac{1}{\mu_p}\frac{\tau_w R_p}{2}\left[1 - h^2 + 2\alpha p(h-1)\right]$$

$$+\frac{1}{\mu_c}\frac{\tau_w R_p}{2}\left[h^2 - \frac{r^2}{R_p{}^2} + 2\alpha_c\left(\frac{r}{p} - h\right)\right] \tag{12}$$

$$Vy = \frac{\tau_w R_p}{2\mu_p}\left[1 - h^2 + 2\alpha p(h-1)\right] \quad + \frac{\tau_w R_p}{2\mu_c}\left[h^2 + (\alpha_c - 2h)\right] \tag{13}$$

In non- dimensional form, we have

$$V_p = 1 - \frac{r^2}{p} + 2\alpha p\left(\frac{r}{R_p} - 1\right) \tag{14}$$

$$V_c = \left[1 - h^2 + 2\alpha_p(h-1) + \frac{\mu_p}{\mu_c}\left\{h^2 - \frac{r_0{}^2}{R_p{}^2} + 2\alpha c\left(\frac{r}{R_p} - h\right)\right\}\right] \tag{15}$$

$$V_y = \left[1 - h^2 + 2\alpha_p(h-1) + \frac{\mu_p}{\mu_c}\left\{h^2 + \alpha_c(\alpha_c - 2h)\right\}\right] \tag{16}$$

Where

$$h = \left(\frac{R_c}{R_p}\right) : \alpha_p = \left(\frac{\tau y_p}{\tau_w}\right), \quad \alpha_c = \left(\frac{\tau_{rc}}{\tau_w}\right), \quad \tau_w = \frac{\partial p}{\partial z}\frac{R_p}{2}, \quad V_i = \left(\frac{V_i}{V_0}\right),$$

$$V_0 = \frac{\tau_w R_p}{2\mu_p}, \quad \text{i=p, c, y}$$

i.e. $V_p = \frac{V_p}{V_o}, \quad V_c = \frac{V_c}{V_0}, \quad V_y = \frac{V_y}{V_o}$.

Volume flow rate is given by

$$Q = Q_1 + Q_2 + Q_3 \tag{17}$$

$$Q_1 = 2\pi \int_{R_c}^{R_p} V_p r dr,$$

$$Q_2 = 2\pi \int_{R_y}^{R_c} V_c r dr$$ and

$$Q_y = 2\pi \int_{0}^{R_y} V_y r dr$$

Now, from equations (11), (12), (13) and (17) we have

$$Q = \frac{\pi \tau_w R_p^{\ 3}}{2\mu_p}\left[\frac{1}{2} - \frac{h^4}{2} + 2\alpha_p \alpha_c^{\ 2} - 2\alpha_p h \alpha_c^{\ 2}\right.$$

$$\left. - \alpha c^2 + \alpha c^2 h^2 + \frac{\mu_p}{\mu_c}\left\{\frac{h^4}{2} - \frac{2}{3}\alpha_c h^3 + \frac{\alpha_c^{\ 4}}{6}\right\}\right] \tag{18}$$

Apparent fluidityis given by

$$F_a = \frac{2Q}{\pi \tau_w R_p^{\ 3}} \tag{19}$$

Using equation (18) in (19)

we get

$$F_a = \frac{1}{\mu_p}\left[\frac{1}{2} - \frac{h^4}{2} + 2\alpha_p \alpha c^2 - 2\alpha_p h \alpha^2{}_c - \alpha_c^{\ 2} + \alpha_c^{\ 2} h^2\right.$$

$$\left. + \frac{\mu_P}{\mu_C}\left\{\frac{H^4}{2} - \frac{2}{3}\alpha_c h^3 + \frac{\alpha_c^{\ 4}}{6}\right\}\right] \tag{20}$$

Putting $h = \left(1 - \frac{d}{R_p}\right)$ we have

$$F_a = 2\frac{d}{R_p}\left(\frac{2 - \alpha_c{}^2 + \alpha_p \alpha_c{}^2}{\mu_p} - \frac{1 - \alpha_c}{\mu_c}\right)$$

$$+ \left(\frac{1}{2} + \frac{\alpha_c{}^4}{6} - \frac{2}{3}\alpha_c\right)\frac{1}{\mu_c} \qquad (21)$$

Result and Discussion -

There are many different models of blood flow through narrow vessels with different boundary conditions and flow parameters but the scope of their quantitative study is very limited because of experimental problems.

Here we have considered $\alpha_p = 0$ and $\alpha_p \neq 0$ i.e. peripheral layer is newtonian and non - Newtonian fluid respectively.

Variation of apparent fluidity with peripheral layer thickness and yield stress (α_c) are given below in tables 1 and 2. Velocity distributions are alos disscussed is Table ...3.......... &......4.........

From fig.1 and 2, it is clear that apparent fluidity fa, increases with d (plasma layer thickness) and decreases with α_c (yield stress). This indicates that when hematocrit decreases, apparent fluidity increases. The results are composed with results of Gupta & Sehadri and found a good similarity in them.Also when α_p=0, fa is found. The values for α_p=0 are greater than those for α_p=0.01 and thus we found that apparent fluidity decreases with yield stress of plasma.

Fig 3 shows that velocity field decreases fastly with r in the peripheral layer. In the core region,

Table - 1

($\mu_p = 1.2$, $\mu_c = 2.2$, $\alpha_c = 0.2$, $\alpha_p = 0$, 0.01)

$\frac{d}{R_p}$	0.08	0.10	0.12	0.14	0.16	0.18
Fa ($\alpha_p = 0$)	0.37	0.421	0.471	0.522	0.558	0.624
Fa ($\alpha_p = .01$)	0.368	0.420	0.470	0.520	0.556	0.623

Variation of Fa with $\frac{d}{R_p}$ (Plasma layer thickness)

Table 2

$\mu_p = 1.2$; $\mu_c = 2.2$; H=40%; $\frac{d}{R_p} = 0.15$; $\alpha_p = 0$, 0.01

α_c	0.16	0.18	0.20	0.22	0.24	0.26
Fa ($\alpha_p = 0$)	0.559	0.553	0.548	0.542	0.537	0.531
Fa ($\alpha_p = 0.1$)	0.558	0.552	0.547	0.541	0.536	0.530

(Varition of apparent fluidity with yield stress)

Table - 3

($h=40\%$, $\alpha / R_p = 0.1$, $\alpha_c = 0.2$, $\alpha_p = 0.01$)

$\frac{r}{Rp}$	0	0.2	0.4	0.6	0.8	1.0
VP ($\alpha p = 0$)	1	0.96	0.84	0.64	0.36	0
VP ($\alpha p = .01$)	0.98	0.94	0.83	0.63	0.35	0

(Variation of Vp with r)

Table - 4

($\mu_c = 2.2$; $\mu_p = 1.2$; $\alpha_c = 0.2$; $h = 0.9,\ 0.85$, $\alpha_p = 0.01$)

$\frac{r}{Rp}$	0	0.2	0.4	0.6	0.8	1.0
$h = .9$ Vc $\alpha_p = 0.0$	0.435	0.457	0.435	0.370	0.261	0.103
$h = .9$ Vc $\alpha_p = 0.1$	0.433	0.455	0.433	0.368	0.259	0.101
$h = .85$ Vc $\alpha_p = 0$	0.486	0.507	0.486	0.421	0.312	0.159
$h = .85$ Vc $\alpha_p = .01$	0.483	0.504	0.483	0.418	0.309	0.156

(Velocity Distribution in core region)

veloity decreases slowly. On comparision of results with enperimental results of Bugliarello and sevilla we found a good similarity. When $\alpha_p = 0$, velocity is greater then those obtained for $\alpha_p = 0.01$.

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CHAPTER VI

A Theoretical Study of Pulsatile Blood Flow Through Narrow Vessels.

6.1 Introduction.

The existing literature dealing with the anamolous characteristics of blood is mainly confined to steady flow. It was believed that in the vascular system, the pulsatile flow damps out by the time it reaches the microcirculation. But recent studies indicate that the pulsatile nature of pressure persists even in capillary bed. The theoretical studies have revealed that the concepts of apparent and relative viscosities are not intrinsic properties of blood but depend on the

interaction of blood and the vessels. hence one may extend the definition of apparent viscosity of steady flow to the oscillatory flow by replacing the steady Poiseuille flow by womersley's oscillatory flux and considering the equality of flow amplitudes.

In present analysis we analyse the pulsatile blood flows throw straight and long narrow rigid circular tubes whose radius is constant. it is assumed that flow of blood is axially symmetric and in the z direction. The blood consist of two layers; one is a plasma of Newtonian fluid and the other is certral core of non Newtonian fluid. The thickness of Plasma layer is assumed to be constant and independent of time and location.

6.2 The Governing Equaations -

The momentum and continuity equations for fully developed viscous incomressible laminar flow under no body forces in the cylindrical coordinates (r, θ, z), whose origin lies on the vessels axis are

$$\rho\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\tau) \quad (1)$$

$$0=\frac{\partial p}{\partial r} \quad (2)$$

$$\frac{\partial v}{\partial z} = 0 \qquad (3)$$

Where V is the axial component of velocity, P the peressure, t the time and ρ is the fluid density.

The pressure gradient which is a function of time but not of axial loction, is given by

$$\frac{\partial p}{\partial z} = P'\{P(t)\} \doteq P'.(1+A \ \mathrm{Sin} \ wt) \qquad (4)$$

Where w is the angular frequency, P' and A are constant.

6.3 Constitutive Equations -

Constitutive equation for the shear stress τ and shear rate e is given by

$$\tau^{\frac{1}{2}} = \tau_0^{\frac{1}{2}} + \mu^{\frac{1}{2}} e^{\frac{1}{2}} \qquad (5)$$

Where τ_0 is the yield stress, μ the coefficient of viscosity. The equation is reduced to that for a Newtonian fluid when $\tau_0 = 0$

The coefficient of viscosityand yield stress of normal blood are given by

$$\mu = \mu_p\left(1 + 0.025H + 7.35x10^{-4} H^2\right) \qquad (6)$$

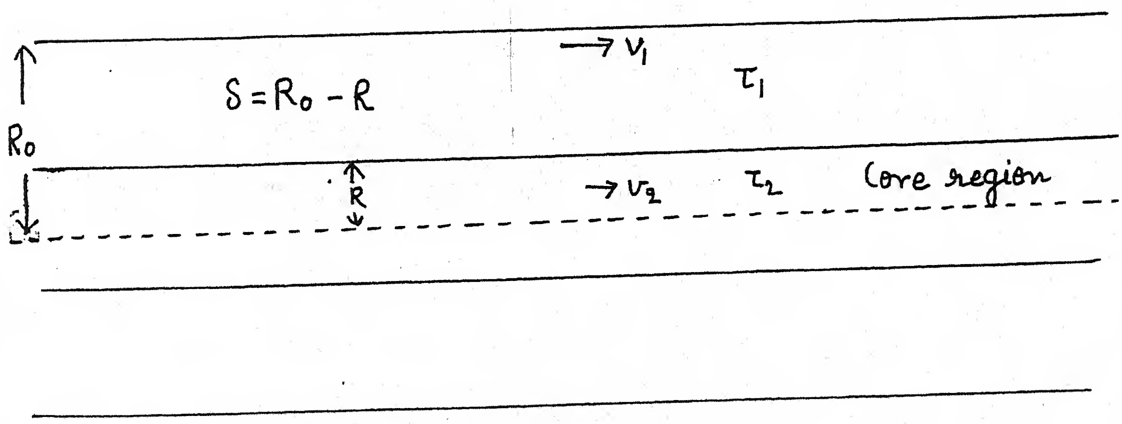
and

$\tau_0 = B(H - H_c)^3$

Where μ_p is the viscosity of plsma, H the hematocrit of blood and H_c is the hemotocrit concentration below which τ_0 is negligible and it normally ranges from 4 to 8. The constant B is strong function of fibrinogen concentration and has value in the range 0.6 $\times10^{-6}$ to 1.2×10^{-6} for normal blood.

These equations are used in numerical results.

6.4 The Boundary Conditions -



It is assumed that the fluid representing the blood contained in a cylindrical tube of radius R_0. consists of two layers - one representing the plasma near the tube wall having a constant thickness δ and the other the central core. The inner radius of the tube has

been taken to be R = X Ro, where X = $1-\dfrac{\delta}{R_0}$

The boundary conditions are given by

$$V_1 = 0 \quad \text{at} \quad r = R_0 \tag{8}$$

$$V_1 = V_2 \quad \text{at} \quad r = xR_0 \tag{9}$$

$$\tau_1 = \tau_2 \quad \text{at} \quad r = xR_0 \tag{10}$$

$$\frac{\partial V_2}{\partial r} = 0 \quad \text{at} \quad r = 0 \tag{11}$$

The quantities in the plasma layer and in the central core are designated by the subscripts 1 and 2, respectively.

6.5 The Mathematical Analysis -

In the mathematical analysis we introduce the following non- dimensional variables and parameters.

$$\bar{r} = \frac{r}{R_0}, \quad \bar{t} = wt, \quad \overline{V_1} = \frac{2\mu_1}{R_o^2 p}, \quad \overline{\tau_1} = \frac{2}{R_o P'}\tau_1 \tag{12}$$

$$\alpha_1^2 = \frac{\rho_1 R_0'^2}{\mu_1} = \frac{\rho_1}{\rho}\alpha^2, \qquad \beta = \frac{2\tau_0}{R_o p'}, \quad \mu_0 = \frac{\mu_2}{\mu_1} \tag{13}$$

Where ρ_1 and ρ are the densities and α_1 and α are the womersley frequency parametere of plasma and blood respectively. The difference between ρ_1 and ρ is so small that we put $\rho_1 = \rho$ in the calculation.

Using equations (12) and (13) in (11) we obtain

the equations of velocity nd motion in plasma and core region.

$$\left|\frac{\partial \overline{Vi}}{\partial \bar{r}}\right| = \bar{\tau}_1 \tag{14}$$

$$\left|\frac{\partial \overline{V_2}}{\partial \bar{r}}\right| = \frac{1}{\mu_0}\left(\bar{\tau}_2^{\frac{1}{2}} - \beta^{\frac{1}{2}}\right)^2 \tag{15}$$

$$\alpha^2 \frac{\partial \overline{V_1}}{\partial \bar{t}} = -2P(\bar{t}) + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\overline{r\tau_1}\right) \tag{16}$$

By making use of the same equations (17) and (18), the boundary conditions are also transfromed into the following forms

$$\overline{V_1} = 0 \quad \text{at} \quad \bar{r} = 1 \tag{17}$$

$$\overline{V_2} = \overline{V_1} \quad \text{at} \quad \bar{r} = x \tag{18}$$

$$\overline{\tau_1} = \overline{\tau_2} \quad \text{at} \quad \bar{r} = x \tag{19}$$

$$\frac{\partial \overline{V_2}}{\partial \bar{r}} = 0 \quad \text{at} \quad \bar{r} = 0 \tag{20}$$

Now in the arterioles the womersley frequency parameter (α or α_1) is very small compared with unity. Then the velocity $\bar{V}$ and shear stress $\bar{\tau}$ are assumed to possess expansions in termes of the parameters α^2, which are of the from -

$$\overline{V_i} = \overline{V_{oi}} + \alpha 2\overline{V_{ii}} \tag{21}$$

$$\bar{\tau}_i = \bar{\tau}_{oi} + \alpha^2 \bar{\tau}_{1i} \tag{22}$$

Terms of order α^4 and of higher orders will be neglected.

6.6 Velocity and shear stress -

The solution of equations (14-16) with boundary conditions (17-19), gives the equations of velocity and shear stress distribution.

Now using the equations (21) and (22) in (16) we get

$$\alpha^2\left[\frac{\partial \overline{V}_{oi}}{\partial \bar{t}} + \alpha^2 \frac{\partial \overline{V}_{ji}}{\partial \bar{t}}\right] = -2\rho(\bar{t}) + \frac{1}{r}\frac{\partial}{\partial r}\left[\bar{r}\left(\bar{\tau}_{oi} + \alpha^2 \bar{\tau} ii\right)\right] \tag{23}$$

Equating the coefficients of α^0 and α^2, we have

$$0 = -2P(\bar{t}) + \frac{1}{r}\frac{\partial}{\partial \bar{r}}\left(\overline{r\tau_{0i}}\right) \tag{24}$$

$$\frac{\partial \overline{V}_{0i}}{\partial \bar{t}} = \frac{1}{\bar{r}}\frac{\partial}{\partial r}\left(\overline{r\tau_{1i}}\right) \tag{25}$$

Integrating equs. (24) & (25) with boundary condition that shear stress is finite at the axis we

obtain-

$$\bar{\tau}_{01} = p(\bar{t})\bar{r} = \bar{\tau}_{02} \tag{26}$$

$$\bar{r}\bar{\tau}_{11} = \int \bar{r}\frac{\partial \bar{V}_{01}}{\partial \bar{t}} d\bar{r} + C_1 \tag{27}$$

$$\bar{r}\bar{\tau}_{12} = \int \bar{r}\frac{\partial \bar{V}_{02}}{\partial \bar{t}} d\bar{r} + C_2 \tag{28}$$

Where C_1 and C_2 are constants.

Similarly, using equ's (21) and (22) in (14) and (15) and equating the coefficients of α^0 and α^2, we have

$$\left|\frac{\partial \bar{V}_{01}}{\partial \bar{r}}\right| = \bar{\tau}_{01} \tag{29}$$

$$\left|\frac{\partial \bar{V}_{11}}{\partial \bar{r}}\right| = \bar{\tau}_{11} \tag{30}$$

$$\left|\frac{\partial \bar{V}_{02}}{\partial \bar{r}}\right| = \frac{1}{\mu_0}\left(\bar{\tau}_{02}^{\frac{1}{2}} - \beta^{\frac{1}{2}}\right)^2 \tag{31}$$

$$\left|\frac{\partial \bar{V}_{12}}{\partial \bar{r}}\right| = \frac{1}{\mu_0}\left\{1 - \left(\frac{\beta}{\bar{\tau}_{02}}\right)^{\frac{1}{2}}\right\}\bar{\tau}_{12} \tag{32}$$

Now putting the values of $\bar{\tau}_{01}$ and $\bar{\tau}_{02}$ from

equation (26) in equ. (29) and (31) and then integrating we have -

$$\bar{V}_{01} = \frac{1}{2} P(\bar{t})\left(1 - \bar{r}^{2}\right) \tag{33}$$

$$\bar{V}_{02} = \frac{1}{2} P(\bar{t})\left(1 - x^2\right) + \frac{1}{\mu_0}\left[\frac{1}{2} p(\bar{t})\left(x^2 - \bar{r}^{2}\right) + \right.$$

$$\left. \beta\left(x - \bar{r}\right) - \frac{4}{3}\beta^{\frac{1}{2}} P^{\frac{1}{2}}(\bar{t})\left(x^{\frac{3}{2}} - \bar{r}^{\frac{3}{2}}\right)\right] \tag{34}$$

From equations (27), (28), (33), (34) we obtain

$$\bar{\tau}_{12} = \frac{dp}{dt}\left[\frac{1}{4}\left(1 - x^2\right)\bar{r} + \frac{1}{\mu_0}\left\{\frac{1}{4} x^2 \bar{r} - \frac{1}{8}\bar{r}^{-3} - \frac{2}{3} y^{\frac{1}{2}} x\left(\frac{1}{2} x^{\frac{3}{2}} \bar{r} - \frac{2}{7}\bar{r}^{-\frac{5}{2}}\right)\right\}\right] \tag{35}$$

$$\bar{\tau}_{11} = \frac{1}{Q\bar{r}}\frac{dp}{d\bar{t}}\left[\left(2\bar{r}^{-2} - \bar{r}^{-4} - x^4\right) + \frac{1}{7\mu_0}\left(7x^4 - 8y^{\frac{1}{2}} x^{\frac{7}{2}}\right)\right] \tag{36}$$

Where $y = \frac{\beta}{|p|}$.

Now using equations (26), (35) and (36) in equatin (22) we get

$$\bar{\tau}_1 = \rho(\bar{t})\bar{r}\left[1 + \alpha^2 \frac{1}{p}\frac{dp}{d\bar{t}} L_1\right] \tag{37}$$

$$\bar{\tau}_2 = P(\bar{t})\bar{r}\left[1+\alpha^2\frac{1}{P}\frac{dp}{dt}L_2\right] \tag{38}$$

where L_1 and L_2 are known functions of ($\bar{r}$, $\bar{t}$), given by

$$L_1 = \frac{1}{8\bar{r}^2}\left[\left(2\bar{r}^2 - \bar{r}^4 - x^4\right) + \frac{1}{7\mu_0}\left(7x^4 - 8y^{\frac{1}{2}}x^{\frac{1}{2}}\right)\right] \tag{39}$$

$$L_2 = \frac{1}{4}\left(1-x^2\right) + \frac{1}{8\mu_0}\left\{\left(2x^2 - \bar{r}^{-2}\right) - \frac{8}{21}y^{\frac{1}{2}}\left(7x^{\frac{3}{2}} - 4\bar{r}^{-\frac{3}{2}}\right)\right\} \tag{40}$$

Shear stress at the wall $\bar{\tau}_w$ is also an important quantity and is given by

$$\bar{\tau}_w = P(\bar{t})\left[1+\alpha^2\frac{1}{p}\frac{dp}{dt}L_3\right] \tag{41}$$

Where

$$L_3 = \frac{1}{8}\left[\left(1-x^4\right) + \frac{1}{7\mu_0}\left(7x^4 - 8y^{\frac{1}{2}}x^{\frac{3}{2}}\right)\right] \tag{42}$$

Now for the velocity distribution, using equations (26), (35), (36) in (30), and applying boundary conditions (17), (18) we have-

$$\bar{V}_{11} = \frac{1}{8}\frac{dp}{dt}\left[\left(\frac{1}{4}\bar{r}^4 - \bar{r}^2 + xLog_e\bar{r} + \frac{3}{4}\right) - \frac{1}{7\mu_0}\left(7x^4 - 8y^{\frac{1}{2}}x^{\frac{7}{2}}\right)Log_e\bar{r}\right] \tag{43}$$

$$\bar{V}_{12} = \frac{1}{8}\frac{dp}{dt}\left[\left(\frac{1}{4}x^4 + x^4 \log_e x - x^2 + \frac{3}{4}\right)\right]$$

$$+\frac{1}{\mu_0}\left\{\frac{1}{3}\left(1-X^2\right)\left(3\left(x^2-\bar{r}^{2}\right)-4y^{\frac{1}{2}}\left(x^{\frac{3}{2}}-\bar{r}^{\frac{3}{2}}\right)\right)-\left(x^4-\frac{8}{7}y^{\frac{1}{2}}x^{\frac{1}{2}}\right)Log_e x\right\}$$

$$-\frac{1}{{\mu_0}^2}\left\{\left(x^2r^2-\frac{1}{4}\bar{r}^{4}-\frac{3}{4}x^4\right)\right\}$$

$$-y^{\frac{1}{2}}\left(\frac{4}{3}\left(x^{\frac{3}{2}}\bar{r}^{2}+x2\bar{r}^{\frac{3}{2}}\right)-\frac{53}{588}\bar{r}^{\frac{7}{2}}\frac{286}{147}x^{\frac{7}{2}}\right)$$

$$+\frac{16}{63}y\left(7x^{\frac{3}{2}}\bar{r}^{\frac{3}{2}}-2\bar{r}^{3}+5x^3\right) \qquad (44)$$

From equations (21), (33), (34), (43) and (44) we obtain

$$V_1 = p(\bar{t})\left[\frac{1}{2}\left(1-\bar{r}^{2}\right)+\alpha^2\frac{1}{p}\frac{dp}{d\bar{t}}N_1\right] \qquad (45)$$

$$\bar{V}_2 = P(\bar{t})\left[N_2+\alpha^2\frac{1}{p}\frac{dp}{d\bar{t}}N_3\right] \qquad (46)$$

Where N_1, N_2 and N_3 are know functions of $(\bar{r},\bar{t})$ given by-

$$N_1 = \frac{1}{8}\left[\left(\frac{1}{4}\bar{r}^{4}-\bar{r}^{2}+x^4 Log_e \bar{r}+\frac{3}{4}\right)\right]$$

$$-\frac{1}{7\mu_0}\left(7x^4-8y^{\frac{1}{2}}x^{\frac{1}{2}}\right)Log_e\bar{r}\Bigg] \qquad (47)$$

$$N_2=\frac{1}{2}\left(1-x^2\right)+\frac{1}{\mu_0}\left\{\frac{1}{2}\left(x^2-\bar{r}^2\right)+y\left(x-\bar{r}\right)-\frac{4}{3}y^{\frac{1}{2}}\left(x^{\frac{3}{2}}-\bar{r}^{\frac{3}{2}}\right)\right\} \qquad (48)$$

$$N_3=\frac{1}{8}\left[\left(x^4Log_e x+\frac{x^4}{4}-x^2+\frac{3}{4}\right)\right]$$

$$+\frac{1}{\mu_0}\left\{\left(1-X^2\right)\left[\left(x^2-\bar{r}^2\right)-\frac{4}{3}y^{\frac{1}{2}}\left(x^{\frac{3}{2}}-\bar{r}^{\frac{3}{2}}\right)\right]\right\}$$

$$-\left(x^4-\frac{8}{7}y^{\frac{1}{2}}x^{\frac{7}{2}}\right)Log_e x$$

$$-\frac{1}{\mu_0^{\,2}}\left\{\left(X^2\bar{r}^2-\frac{1}{4}\bar{r}^4-\frac{3}{4}x^4\right)\right\}$$

$$-\frac{4}{3}y^{\frac{1}{2}}\left(X^{\frac{3}{2}}\bar{r}^2+X2\bar{r}^{\frac{3}{2}}-\frac{159}{2352}\bar{r}^{\frac{7}{2}}-\frac{143}{98}X^{\frac{7}{2}}\right)$$

$$+\frac{16}{63}y\left(7x^{\frac{3}{2}}\bar{r}^{\frac{3}{2}}-2\bar{r}^3+5X^3\right) \qquad (49)$$

6.7 Flow Rate -

the volume flow rate Q (t) of the fluid flowing per unit time across the cross section of the vessel,

is given by -

$$Q(t) = \int_{r=0}^{r=xR_0} 2\pi V_2 r dr + \int_{r=XR_0}^{r=R_0} 2\pi V_1 r dr$$

$$= \frac{\pi R_0{}^4 P^1}{\mu_1}\left[\int_0^X \bar{V}_2 \bar{r} d\bar{r} + \int_x^1 \bar{V}_1 \bar{r} d\bar{r}\right] \qquad (50)$$

Now, using equations (45) and (46) in (50) we obtain

$$\bar{Q}(\bar{t}) = \frac{Q(t)}{Q_1} = p(\bar{t})\left[5 + \alpha^2 \frac{1}{p}\frac{dp}{d\bar{t}} T\right] \qquad (51)$$

Where

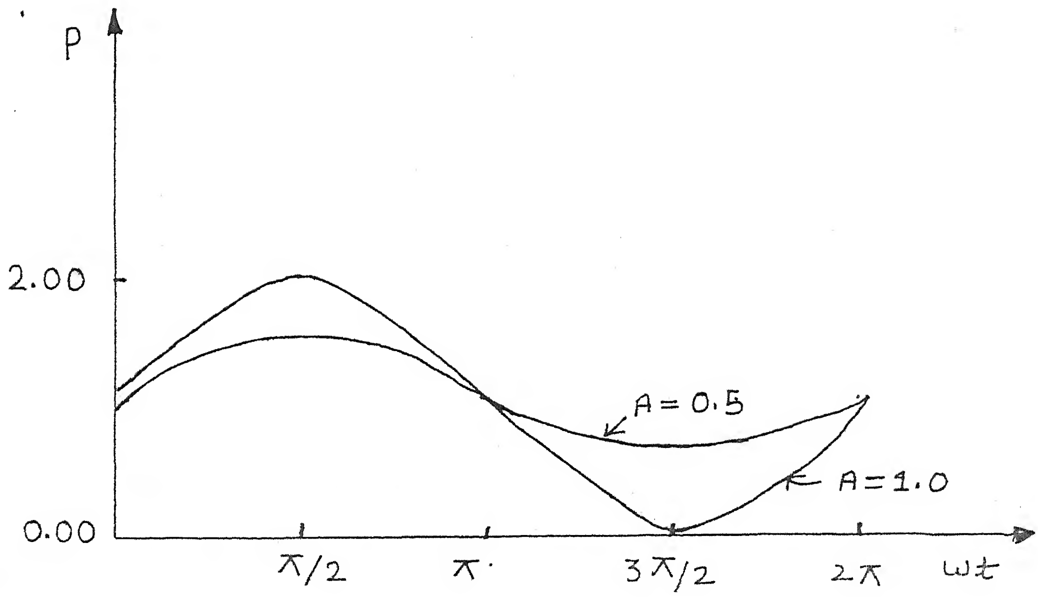
$Q_1 = \dfrac{\pi R_0{}^4 P^1}{8\mu_1}$, S and T are known functions of $\bar{t}$, given by

$$S = 1 - X^4 + \frac{X^3}{21\mu_0}\left(21X - 48y^{\frac{1}{2}}x^{\frac{1}{2}} + 28y\right) \qquad (52)$$

$$T = \frac{1}{6}\left(1 - 3X^4 + 2X^6\right) + \frac{X^{\frac{1}{2}}}{7\mu_0}\left(7x^{\frac{1}{2}} - 8y^{\frac{1}{2}}\right) Log_e X$$

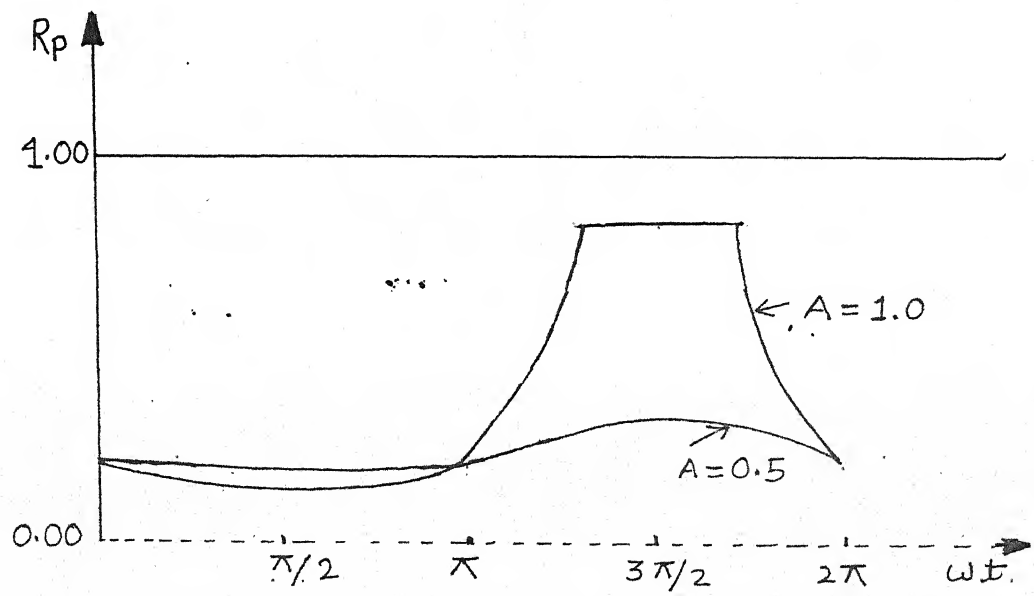
$$+ \frac{X^5}{\mu_0{}^2}\left(\frac{1}{6}X + \frac{479}{462} y^{\frac{1}{2}} X^{\frac{1}{2}} + \frac{3}{35} y\right) \qquad (53)$$

The steady flow rate can be obained by substituting p (t)=1 in equ. (51).



VARIATION OF REDUCED PRESSURE WITH REDUCED AMPLITUDE

Figure 6·1



VARIATION OF REDUCED PLUG RADIUS WITH REDUCED AMPLITUDE

Fig 6.2

Results and Discussions -

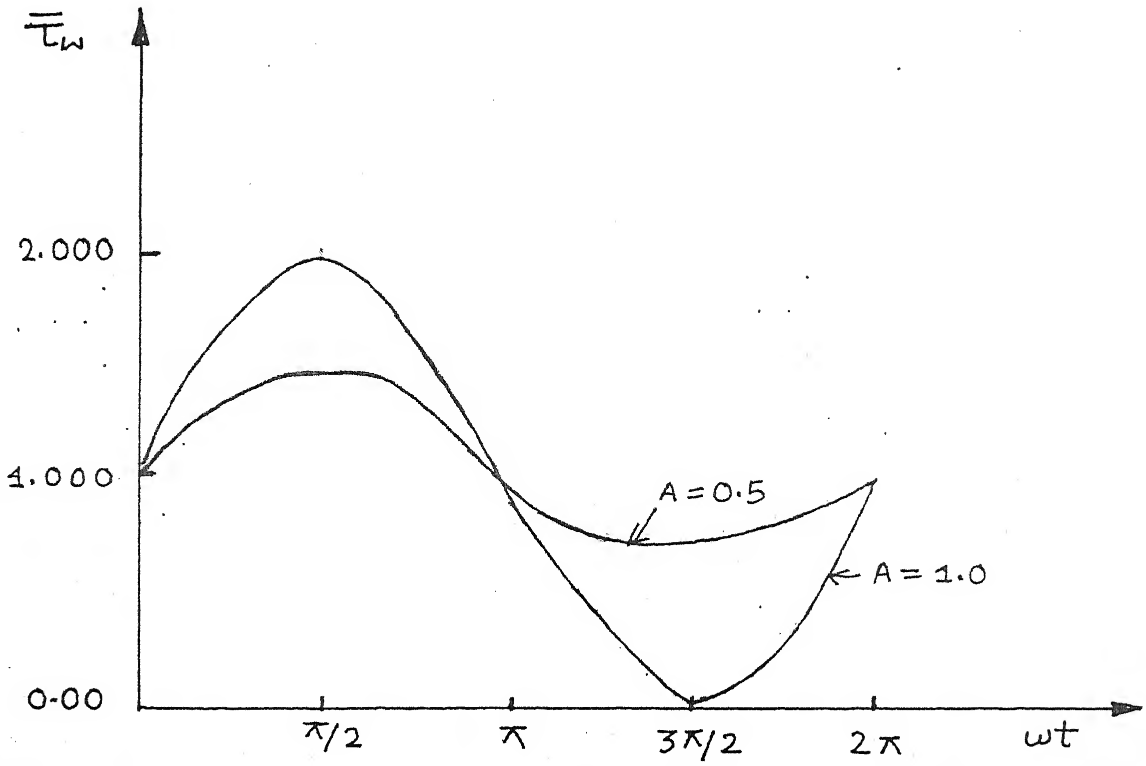
The characteristies discussed are the pressure, shear sress at the wall, flow rate and visocsity.

Here we have considred the case of vessel radius $R_0 = 10x10^{-3}$ cm, mean pressure gradient P' = 0.62 x 10^4 dynes/cm^3 of which plasma viscosity is μ_1=0.012 centipose.

The value of womrersely frequency parameter α is about 0.1 in arterioles. The experimentally measured values of viscosity of normal human blood varies from 0.03 to 0.05 Poise. The value of yield stress for normal blood is of order 0.04 dynes per cm^2. These values depend on various factors such as, hematocrit, fibrinogen concentration etc.

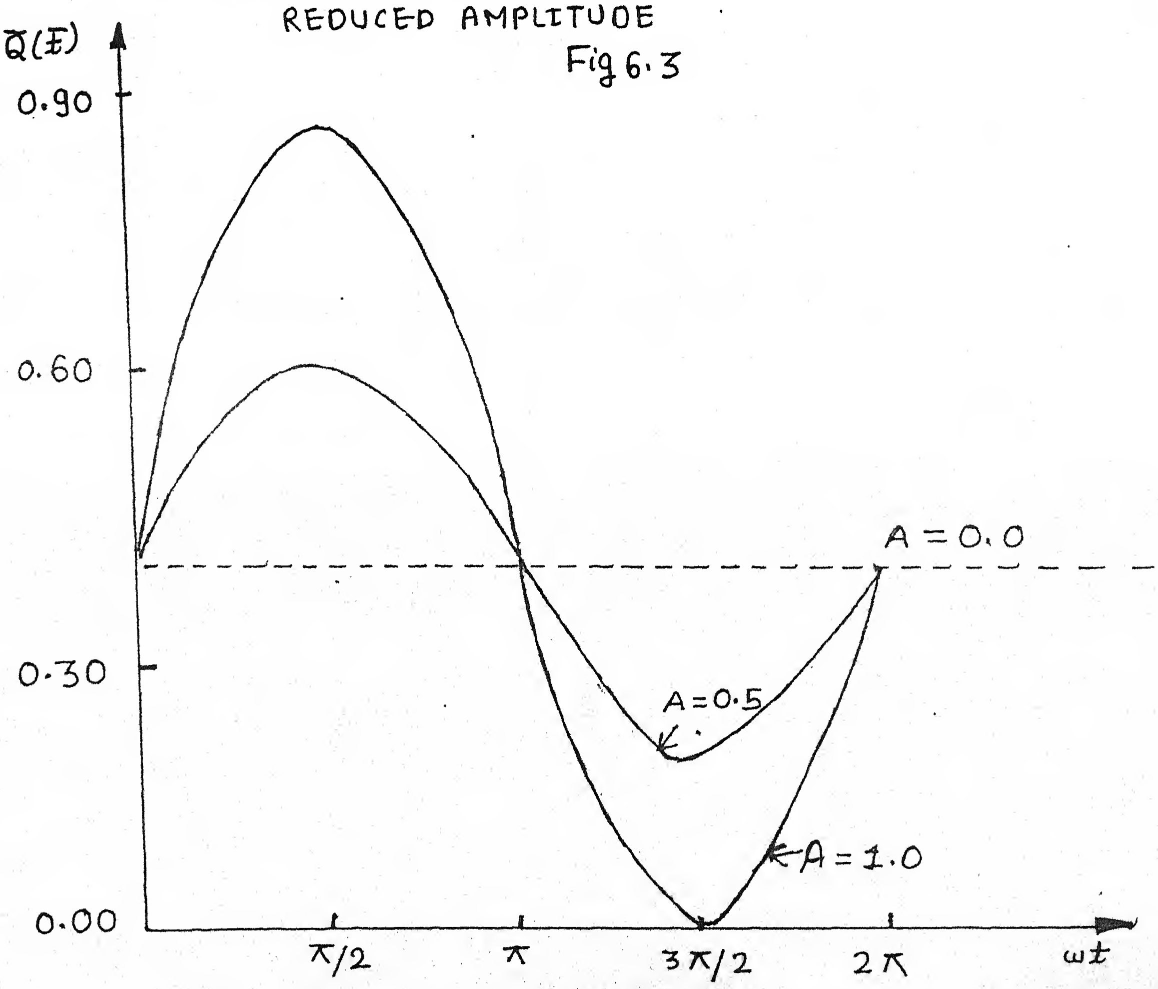
It is seen that the effect of α on velocity, shear stress, plug radius and flow radius is very small in the range $0 \le \alpha \le 1$.

Fig. 6.1 shows the effect of reduced amplitude A of pressure gradient on pulsatile blood flow.It is seen that the values of R_p, $\bar{\tau}_w$ and $\bar{Q}(t)$ vary greatlywith respect to time specially for large values of A.



VARIATION OF REDUCED WALL SHEAR STRESS WITH REDUCED AMPLITUDE

Fig 6.3



VARIATION OF REDUCED FLOW RATE WITH REDUCED AMPLITUDE

Fig 6.4

The influence of reduced y\ield stress on R_p, $\bar{\tau}_w$ and $\bar{Q}(\bar{t})$ are shown in the fig 6.2. Due to the presence of plasma layer the value of $\bar{\tau}_w$ is not affected very much. The blood structure changes when the value of yield stress changes. The flow rate for steady flow can be seen from fig 6.2. It is seen that flow rate increases gradually with the increase in the thickness of plasma layer. For smaller values of $\frac{\delta}{R_0}$ the flow rate in central core is manimum and grdually decreases when the thickness of plasma layer is increased.

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Chapter VII

Theoretical Model of Steady Blood Flow in Narrow Vessel

7.1 Introduction :

Mathematical modelling of blood flow in different vessels under various physiological conditions have drawn the attention of several workers. Since poiseuilles empirical euqation of finding out the apparent viscosity of viscous fluid in fine glass capillaries, many workers attempted to find the apparent viscosity invivo and invitro, which is an important parameter affecting the blood rheology. Fahraeus and Lindquist found that apparent viscosity of blood decreases with the decrease of tube diameter below 500 μm down to 7 μm. Recently, Kiani and Hudetz (1991) developed the model assuming no yield stress behaviour of blood in these vessels. It has been recognised by several experimental observations and viscometric datas that blood possesses considerable amount of yield stress in narrow vessels. Scott Blair (1959), Charm and Kurland (1968), bugliarello and Sevilla (1970), Oba (1981, 89) have proposed casson constitutive equation to analyse the blood behaviour in narrow vessels at low and high shear rates. In the present analysis we have assumed that the fluid in core region satisfy casson equation and in marginal layer region satisfy a Newtonian equation. Apparent viscosity of blood has been determined as a function of yield stress, vessel diameter and peripheral layer (PPL) thickness.

7.2 *Mathamatical Analysis :*

Incompressible fully developed laminar flow of blood is considered in a circular tube of small diameter. Two-layer flow model is assumed. Core fluid follow the casson equation.

$$\tau^{1/2} = \tau_y^{1/2} + \mu^{1/2}\,\dot{\gamma}^{1/2} \tag{1}$$

and plasma peripheral layer follows the Newtonian equation

$$\tau = \mu_p\,\dot{\gamma} \tag{2}$$

Where τ is shear stress, τ_y the yield stress, $\dot{\gamma}$ the shear rates, μ_c casson core viscosity, μ_p Newtonian plasma viscosity.

Steady incompressible laminar flow equation

$$\frac{\partial P}{\partial Z} = \frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rz}) \tag{3}$$

after integration yields

$$\tau_{rz} = \frac{pr}{2L} \tag{4}$$

Where

z, r = axial and radial measurements

τ_{rz} = shear stress normal to r in z direction

p = pressure drop along length L of the tube.

At the interfacial boundary between core and marginal fluids, shear stresses are equal and at the wall surface noslip condition are assumed.

Substituting (1) and (2) in (4) we get

$$V_p = \frac{P}{4L\,\mu_p}(Rw^2 - r^2); \quad R_c \le r \le Rw \tag{5}$$

$$V_c = \frac{P}{4L\mu_c}(R_c^2 - r^2) - \frac{4}{3}\left(\frac{P\tau y}{2L}\right)^{1/2}\frac{1}{\mu_c}(R_c^{3/2} - r^{3/2})$$

$$+ \frac{\tau_y}{\mu_c}(R_c - r) + \frac{P}{4L\mu_p}(R_w^2 - R_c^2) \quad ; 0 \le r \le Rc \tag{6}$$

Where V_p denotes plasma velocity and Vc denotes core velocity.

Plug velocity is obtained by putting $r=R_c$ in equation (6) as

$$V_1 = \frac{P}{4L\mu_c}(R_c^2 - R_1^2) - \frac{4}{3\mu_c}\left(\frac{P\tau_y}{2L}\right)^{1/2}(R_1^{3/2} - R_1^{3/2}) + \frac{\tau_y}{\mu_c}(R_c - R_1)$$

$$+ \frac{P}{4L\mu_p}(R_w^2 - R_c^2) \tag{7}$$

The total flow rate $Q = Q_p + Q_c + Q_1$

$$\text{or } Q = \int_{R_c}^{R} 2\pi r dr \cdot V_p + \int_{R_1}^{R_c} 2\pi r dr \cdot V_c + \pi R_1^2 \cdot V_1$$

$$\text{or } Q = \frac{P\pi R_w^4}{8L\mu_p}\left[1 - \left(\frac{R_c}{R_w}\right)^4 + \left\{\left(\frac{\mu p}{\mu_c}\right)\frac{R_1^4}{R_w} - \frac{16}{7}\left(\frac{R_1}{R_w}\right)^{7/2} \cdot \left(\frac{R_1}{R_w}\right)^{1/2}\right.\right.$$

$$\left.\left. + \frac{4}{3}\left(\frac{R_c}{R_w}\right)^3\left(\frac{R_e}{R_w}\right) - \frac{1}{21}\left(\frac{R_e}{R_w}\right)^4\right\}\right] \tag{8}$$

Where plug radius $R_1 = \frac{2L\tau_y}{P}$

The apparent blood viscosity μ_{app} is obtained as

$$\mu_{app} = \mu_p\left[\left\{1 - \left(1 - \frac{2\delta}{d}\right)^4\right\} + \frac{\mu_p}{\mu_c}\left\{\left(1 - \frac{2\delta}{d}\right)^4 - \frac{16}{7}\left(1 - \frac{2\delta}{d}\right)^{7/2} \cdot \alpha^{1/2}\right.\right.$$

$$\left.\left. + \frac{4}{3}\left(1 - \frac{2\delta}{d}\right)^3 \alpha - \frac{1}{21}\alpha^4\right\}\right]^{-1} \tag{9}$$

Where

$$\alpha = \frac{R_1}{R_w}, \quad \delta = R_w - R_c, \quad d = 2R_w$$

Upto first order smallness of δ, equation (9) reduces to

$$\mu_{app} = \mu_p\left[\frac{\mu_p}{\mu_c}F(\alpha) + \frac{8\delta}{d}\left(1 - \frac{\mu_p}{\mu_c}\phi(\alpha)\right)\right]^{-1} \tag{10}$$

Where $F(\alpha) = 1 - \frac{16}{7}\alpha^{1/2} + \frac{4}{3}\alpha - \frac{1}{21}\alpha^4$

$$\phi(\alpha) = 1 - 2\alpha^{1/2} + \alpha$$

As α is a measurement of yield stress, therefore at high shear rates it takes small value but not zero as chosen by Kiani and Hudetz (1991). For

constant $F(\alpha)$ and $\phi(\alpha)$, apparent viscosity decreases with the increase in δ. Fahraeus Lindquist effect can be observed from equation (10). Charm and Kurland have shown that δ increases with decreasing α. We have taken the datas of Charm and Kurland and shown the variation of μ_{app} with α at constant $\frac{\delta}{d}$ in table I. Assuming that the casson viscosity is related by general equation

$$\mu_c = \mu_p (1-KH)^{-1} \qquad (11)$$

Where H is a function of hematocrit and K is an experimental constant, we can obtain μ_{app} at different concentration of hematocrit.

TABLE 9

[Charm and Kurland (1968), μ_p = .0125 poise, μ_c = .0259 pose, $\frac{\delta}{d}$ = .01, H = .448]

μ_{app}	α	$F(\alpha)$	$f(\alpha)$
2.7790	.0005	.9495	.9559
2.8293	.0010	.9291	.9378
3.0584	.0050	.8451	.8636

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